Variation of "Physics Constants" over Time

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In 1937, Dirac suspected that most physics constants were truly variable parameters in slow evolution over time [1]. *After the publication of Dirac, several other physicists emitted similar assumptions* [18,19]. *Others were convinced that at least some constants, such as the universal gravitational constant, could change over time* [19, 20, 21].

In 1929, Hubble observed that the universe is expanding [2]. *In a model of the universe that we have developed, the matter and the photons that make up the universe are moving away from the center of mass of the universe* [3]. *This has the effect of slightly increasing the refractive index of the vacuum and slowly increase the speed of light over time* [3]. *The variations are slow and well below the detection threshold for the current measuring devices. According to our model, the speed of light is not the only change over time. Several other parameters would do the same. Nevertheless, a few parameters really deserve the title of "constants".*

KEY WORDS: Dirac, physics constants, expansion of the universe

1. INTRODUCTION

Physics constants are very useful in various equations to describe the universe and the phenomena around us. These values are useful because of their constancy over time. But are they really?

Some "constants" are actually physics constants for purely geometrical reasons. Other constants are due to the fact that they come from a ratio of values having the same units. In such a ratio, the percentage change of the numerator over time is identical to the denominator. Both variations therefore cancel and the resulting report becomes constant. However, other fundamental constants of physics vary slightly and slowly over time due to the expansion of the universe.

We want to warn the reader that to define the physics constants with respect to the speed of light in vacuum *c* has the effect of giving the impression that the constants do not vary over time. Indeed, making a metaphor, if several race cars accelerate at the same rate while being side by side and if drivers use either of the cars as a reference point, they could momentarily lose sight of that in reality, they are accelerating compared to the race track. In our case, if the light accelerates over time and if the speed of light is taken as reference for the units of length, time, etc., it could happen that no one finds variation over time. The variable parameters over time could then look like real constants, without actually being.

When the time comes to set up a universally recognized system of measures the speed of light remains, for now, the best option. Even though we now know that light accelerates slowly over time. So, deciding, knowingly, to maintain the value of the speed of light constant to facilitate the creation of measurement standards can be useful.

However, just like car drivers, we must not lose sight that light accelerates. If a racing driver is accelerating on a race track and if he is thinking that he is stopped, he will have a hard time explaining why all the scenery moves around him. In the same vein, thinking that light does not vary over time could leads to phenomena difficult to explain in a reasonable manner.

In this document, **considering that light accelerates over time**, we will focus on one at a time to determine the effects of the expansion of the universe in what we commonly consider as "constants" of physics. We will provide, in conclusion, a summary of the values and the variations associated with each of the physical constants that we analyzed.

2. DEVELOPPEMENT

In this chapter, we will list the various physics constants and find the variations over time for each of them. As some constants actually come from more fundamental constants, it can be difficult to treat some cases at the beginning. Therefore, we must find first the most fundamental constants, those that are really constant over time, and deal with them first. Next, we will treat the constants that have only one unknown value at the time. We must therefore follow a logical order to do this exercise. It would not be allowed to do so, beginning randomly.

2.1. Variation of the Constant *N* over Time

The number N comes from the large numbers hypothesis of Dirac [22]. If we associate a mass m_{ph} to a photon that has a wavelength equal to the apparent circumference of the universe $2\cdot\pi\cdot R_u$, the value of the number N corresponds to the number of photons that can be contained in the apparent mass of the universe m_u [4]. We will show here that the number N is constant. The number N is a unit-less number defined as being:

$$N = \frac{m_u}{m_{ph}} \approx 6.3\times 10^{121} \tag{1}$$

Without being accurate, Dirac reported in his large numbers hypothesis [22], a number N that is around 10^{120}. In his book "The Thermodynamic Universe", Sidharth lists several relationships that lead approximately to that number. However, in a work that we have already presented in the past, we list several relationships leading to the number N that we can consider accurate [4].

Einstein has shown that a mass m can be converted to pure energy (photons) with the famous equation [5,6]:

$$E = m \cdot c^2 \tag{2}$$

Here, c is the speed of light in vacuum. According to the CODATA 2010 [8], $c \approx 299792458$ m/s.

This equation is bidirectional. Pushing the argument further, this means that the material is somehow made of photons. Whatever the arrangement and the methods used to confine the photons to obtain the various constituents of matter, this means that the most fundamental particle of matter is simply confined light.

If we accept the assumption that all masses in the universe are made up of photons and that they will go through all the same changes over time, the ratio N is necessarily constant since the variations of the masses will cancel between the numerator and the denominator in the equation **(2)**.

$$N = \text{constant} \tag{3}$$

As well as being constant, the number N, if one day it is fully discovered, should be a positive even integer composed of 122 digits. Indeed, if any particle is born in vacuum, it is born with its twin which will counterbalance the energy and the momentum so that the total energy and momentum of the system (the whole universe) is equal to zero .

2.2. Variation of the Fine Structure Constant over Time

We will show here that the fine structure constant is really is constant over time.

We have already shown that the constant N can also be associated to the fine structure constant α by the following equation [7]:

$$N = \frac{1}{\alpha^{57}} \approx 6.30341951(12) \times 10^{121} \tag{4}$$

According to the CODATA 2010 [8], $\alpha \approx 7.2973525698(24) \times 10^{-3}$.

As N is constant over time due to the fact that it represents a dimensionless ratio, we conclude that the value of α is constant over time and is not influenced by the process of expansion of the universe. The fine structure constant α has no unit and, like N, it is a ratio and any changes applied to the numerator associated with it would also be applied to the corresponding denominator, so that α is a real constant over time .

$$\alpha = \text{constant} \tag{5}$$

2.3. Variation of the Hubble "Constant" H_0 over Time

Thanks to his observations, Edwin Powell Hubble showed in 1929 that the universe is expanding [2]. He found that the galaxies, regardless of their own movements, move away from each other to even greater speeds if they were far away from each other. He deduced a law and a parameter that he called the "Hubble constant H_0".

As the apparent age of the universe T_u is linked to the inverse of the Hubble constant, [10] the Hubble constant H_0 is not really a constant. It seems constant to our time scale due to the fact that the universe is relatively old. Let's try to determine the annual variation in the Hubble constant ΔH_0.

The apparent age of the universe T_u is defined as following:

$$T_u = \frac{1}{H_0} \tag{6}$$

We have already shown that the present value of the Hubble constant H_0 could be described as a function of the fine structure constant α, the speed of light c, the classical radius of the electron r_e and β as following [11]:

$$H_0 = \frac{c \cdot \alpha^{19} \cdot \beta^{1/2}}{r_e} \approx 72.09548632(46)\ \text{km/(s} \cdot \text{MParsec)} \tag{7}$$

According to the CODATA 2010 [8], $r_e \approx 2.8179403267(27) \times 10^{-15}$ m.

The value of β is irrational. It gives the ratio between the expansion speed of the material universe and the speed of light in vacuum c [3] :

$$\beta = 3 - \sqrt{5} \approx 0.764 \tag{8}$$

This β constant comes from a document where we assumed that the speed of light is constantly increasing (currently equal to c) over time and will tend towards an asymptotic k value when the apparent radius of curvature of the universe R_u will tend to infinity. To know all the paths that has been followed, we refer you to the paper that is available on the Internet [3].

The value of the k constant is given by:

$$k = c \cdot \sqrt{2+\sqrt{5}} \approx 6.17024151 \times 10^{8} \text{ m/s} \qquad \textbf{(9)}$$

The value of the Hubble constant H_0 obtained in equation **(7)** is supported by the value measured by the of Wang Xiaofeng team [14] who obtained $H_0 \approx 72.1 \pm 0{,}9$ km/(s·MParsec).

After a year, the apparent age of the universe will be given as a function of H_0' :

$$\frac{1}{H_0'} = T_u + 1\,\text{year} \approx \frac{1}{H_0} + 31557600\text{ s} \qquad \textbf{(10)}$$

Currently, the annual variation of H_0 may be evaluated by ΔH_0 :

$$\Delta H_0 = \frac{H_0' - H_0}{1\,\text{year}} \approx -5.3 \times 10^{-9} \frac{\text{km}}{\text{s} \cdot \text{MParsec} \cdot \text{year}} \qquad \textbf{(11)}$$

According to equation **(11)**, we could improve the value of the Hubble constant H_0 of the equation **(7)** by a factor of about 87.

2.4. Variation of the Apparent Radius of curvature R_u and r_u of the Universe over Time

If the universe is expanding, it is normal to consider that the apparent radius of the luminous universe R_u increases over time. Let's try to evaluate its annual variation.

Currently, our best evaluation of R_u is based on the knowledge of the Rydberg constant R_∞. It is the most accurate constant. According to CODATA 2010 [8], the value of R_∞ is:

$$R_\infty = \frac{m_e \cdot q_e^4}{8 \cdot \varepsilon_0^2 \cdot h^3 \cdot c} \approx 10973731.568539(55)\text{ m}^{-1} \qquad \textbf{(12)}$$

According to the CODATA 2010 [8] :

- The mass of the electron $m_e \approx 9.10938291(40) \times 10^{-31}$ kg
- The charge of the electron $q_e \approx 1.602176565(35) \times 10^{-19}$ c

- The permittivity of vacuum $\varepsilon_0 \approx 8.854187817 \times 10^{-12}$ F/m
- The Planck constant $h \approx 6.62606957(29) \times 10^{-34}$ J·s

Usually, the apparent radius of the luminous universe R_u (sometimes called the Hubble radius or the dimension of the universe) is described as following [15,16,17]

$$R_u = \frac{c}{H_0} \tag{13}$$

In previous work, we have shown that the apparent radius of curvature of the luminous universe R_u may be defined as following [9] :

$$R_u = \frac{1}{4 \cdot \pi \cdot R_\infty \cdot \beta^{1/2} \cdot \alpha^{16}} \tag{14}$$

$$R_u \approx 1.2831078806(68) \times 10^{26}\,\text{m} \tag{15}$$

This method of calculation for R_u is currently the most precise of all [9].

Currently, the speed of light in vacuum is c. Neglecting the variation undergone by light in vacuum over time [25], the annual variation in the apparent radius of the universe ΔR_u will be (considering that 1 year ≈ 31 557 600 seconds):

$$\Delta R_u \approx c \cdot 31557600\ \text{s/year} \approx 9.5 \times 10^{15}\ \text{m/year} \tag{16}$$

We conclude that it is still possible to improve our estimate of the apparent radius of the universe by a factor of about 72. Beyond that, the annual variation of this cosmological parameter will make change the last digit annually.

The value of the apparent radius of curvature of the material universe r_u is:

$$r_u = \beta \cdot R_u \approx 9.802071983(52) \times 10^{25}\,\text{m} \tag{17}$$

The annual variation of the apparent radius of curvature of the universe Δr_u is:

$$\Delta r_u = \beta \cdot \Delta R_u \approx 7.2 \times 10^{15}\ \text{m/year} \tag{18}$$

2.5. Variation of the classical radius of the electron r_e over Time

Although most physicists now agree that the electron is in reality point-like (we think that it is probably of the Planck length l_p magnitude), the classical radius of the electron r_e represents the radius of the sphere for which all the electromagnetic energy of the electron is equal to the mass energy of the electron m_e (according to classical electrodynamics).

According to the CODATA 2010 [8], the classical radius of the electron is $r_e \approx 2.8179403267(27) \times 10^{-15}$ m.

In a previous work [9], we have already shown that the classical radius of the electron r_e was linked to the apparent radius of the universe R_u as following:

$$R_u = \frac{r_e}{\beta^{1/2} \cdot \alpha^{19}} = \frac{r_e \cdot N^{1/3}}{\beta^{1/2}} \tag{19}$$

Isolating r_e, we obtain:

$$r_e = \frac{R_u \cdot \beta^{1/2}}{N^{1/3}} \tag{20}$$

We note that, on the right side of the equation, N and β are constant over time. Consequently, the classical radius of the electron r_e must increase in the same proportions then the apparent radius of curvature of the universe R_u overt time. The annual variation of the classical radius of the electron Δr_e then be:

$$\Delta r_e \approx \frac{r_e}{R_u} \cdot \Delta R_u \approx 2.1 \times 10^{-25} \text{ m/year} \tag{21}$$

2.6. Variation of the Speed of Light *c* over Time

To calculate the annual variation of the speed of light, let's begin with the equation **(7)**. We can show that the annual variation ΔH_0 is given by the following equation :

$$\Delta H_0 = \left[\frac{(c + \Delta c \cdot 31557600 \text{ s})}{r_e + \Delta r_e \cdot 31557600 \text{ s}} - \frac{c}{r_e} \right] \cdot \frac{\alpha^{19} \cdot \beta^{1/2}}{1 \text{ year}} \tag{22}$$

From another side, we also know that ΔH_0 is equal to:

$$\Delta H_0 = \frac{H_0}{(1 + H_0 \cdot 31557600 \text{s}) \cdot 1 \text{ year}} \tag{23}$$

Let's make equal the equations **(22)** and **(23)**. Let's then isolate Δc. Using the equation **(7)** and doing a few approximations, we then get:

$$\Delta c = \frac{c \cdot \Delta r_e}{r_e} \approx \frac{0.022\,\text{m/s}}{\text{year}} \tag{24}$$

Thanks to the theory of the relativity, Einstein has show that the presence of an important mass was changing the index of refraction n of the surrounding vacuum. His firs paper on that subject has been written in 1911 and it was based on the special relativity. However, based on the general relativity, he must multiply the variation of the refractive index by 2. His theory has then been confirmed by the discovery of the existence of the gravitational lenses.

According to a model of the universe that we presented in previous works, regardless of the dispersion of matter in it, the fact that the universe is expanding forces us to move away from a center of mass. With a Schwarzschild modified metric, we have shown that the index of refraction decreases and allows a slight increase of the speed of light over time.

The annual variation Δc of the equation **(24)** corresponds to the variation of the speed corresponding to the acceleration of light $a_L = c{\cdot}H_0$ on the outer edge of the luminous universe [3] (if we convert the units in m/(s·year)):

$$\Delta c = c \cdot H_0 \cdot 31557600\,\text{s/year} \approx \frac{0.022\,\text{m/s}}{\text{year}} \tag{25}$$

The Pioneer effect [3] is caused by the acceleration of light over time. Indeed, the Pioneer 10/11 probe speeds are determined using the Doppler effect, which is based on Einstein's special relativity. As Einstein assumed, wrongly (we believe), that the speed of light was constant over time, the Doppler effect equations use the constant c regardless of the variation in the speed of light over time. This leads to conclude, wrongly, that the probes slow down over time. In fact, the probes do not undergo any deceleration. The Pioneer effect is an illusion caused by the use of an equation (Doppler effect) that do not account for the acceleration of light over time [3].

2.7. Variation of the Permeability of the Vacuum μ_0 over Time

The speed of light in vacuum c is a function of the permeability of the vacuum μ_0 and of the permittivity of the vacuum ε_0 , since:

$$c = \sqrt{\frac{1}{\mu_0 \cdot \varepsilon_0}} \tag{26}$$

The physicists had the choice between varying the permeability of the vacuum μ_0 or to considering it as being fixed and then varying the permittivity of the vacuum ε_0. They choose to consider the permeability μ_0 as being constant and to attribute this following value to it:

$$\mu_0 = 4 \cdot \pi \times 10^{7}\ \mathrm{T \cdot m/A} \tag{27}$$

The annual variation of the annual variation is therefore zero.

2.8. Variation of the Vacuum Permittivity ε_0 over Time

From the equation **(26)**, we can isolate the vacuum permittivity constant ε_0 :

$$\varepsilon_0 = \frac{1}{\mu_0 \cdot c^2} \approx 8.85418782 \times 10^{-12}\ \mathrm{A}^2 \cdot \mathrm{s}^4 / \left(\mathrm{kg} \cdot \mathrm{m}^3\right) \tag{28}$$

So, the annual variation in the vacuum permittivity constant $\Delta\varepsilon_0$ will be:

$$\Delta\varepsilon_0 = \frac{1}{\mu_0 \cdot 1\,\mathrm{year}} \cdot \left(\frac{1}{\left(c + \Delta c \cdot 1\,\mathrm{year}\right)^2} - \frac{1}{c^2} \right) \tag{29}$$

If Δc is calculated for the periphery of the luminous universe, $\Delta\varepsilon_0$ is:

$$\Delta\varepsilon_0 \approx -1.3 \times 10^{-21}\ \mathrm{A}^2 \cdot \mathrm{s}^4 / \left(\mathrm{kg} \cdot \mathrm{m}^3 \cdot \mathrm{year}\right) \ \text{(for luminous universe)} \tag{30}$$

2.9. Variation of Vacuum Impedance Z_0 over Time

The vacuum impedance Z_0 may be described as a function of the vacuum permeability μ_0 and the speed of light c.

$$Z_0 = \mu_0 \cdot c \approx 376.730313\,\Omega \tag{31}$$

As shown above, the vacuum permeability μ_0 is constant, but the speed of light varies over time. So it's the same with the vacuum impedance.

The annual variation in the vacuum impedance will be ΔZ_0:

$$\Delta Z_0 = \mu_0 \cdot \Delta c \tag{32}$$

On the borders of luminous universe, ΔZ_0 is:

$$\Delta Z_0 \approx 2.8 \times 10^{-8}\ \Omega/\mathrm{year} \tag{33}$$

However, for the material universe, the annual variation of the vacuum impedance ΔZ_0 is:

$$\Delta Z_0 \approx 3.6\times 10^{-8}\ \Omega/\text{year} \tag{34}$$

2.10. Variation of the Apparent Mass of the Universe m_u over Time

The apparent mass of the universe is usually defined as following [12,13] :

$$m_u = \frac{c^3}{G\cdot H_0} \tag{35}$$

Thanks to the equations **(7)** and **(44)**, it may be defined as a function of the mass of the electron m_e, the fine structure constant α and β as following [7]:

$$m_u = \frac{m_e\cdot \beta^{1/2}}{\alpha^{39}} \approx 1.72809823(8)\times 10^{53}\ \text{kg} \tag{36}$$

Because of the principle of conservation of energy, we know that the overall energy of the universe E_u remains constant over time.

$$E_u = m_u\cdot c^2 = \text{constant} \tag{37}$$

It follows that the apparent mass of the universe can be determined as following:

$$m_u = \frac{E_u}{c^2} \tag{38}$$

As the speed of light c varies over time, we can determine the annual variation in the apparent mass of the universe like this:

$$\Delta m_u = \frac{E_u}{1\,\text{year}}\cdot\left(\frac{1}{(c+\Delta c\cdot 1\,\text{year})^2}-\frac{1}{c^2}\right)\approx \frac{m_u\cdot c^2}{1\,\text{year}}\cdot\left(\frac{1}{(c+\Delta c\cdot 1\,\text{year})^2}-\frac{1}{c^2}\right) \tag{39}$$

$$\Delta m_u \approx -2.5\times 10^{43}\ \frac{\text{kg}}{\text{year}} \tag{40}$$

This is huge since the loss of mass is equivalent, annually, to about 13 trillion times the mass of our sun.

2.11. Variation of the Mass of Photons m_{ph} over Time

We define the mass m_{ph} as being the one that is associated to the lowest energy photon, that is to say the one that has a wavelength equal to the apparent circumference of the universe $2\pi \cdot R_u$.

From the equations **(1)**, **(4)** and **(36)**, we obtain the precise value of m_{ph}:

$$m_{ph} = \alpha^{18} \cdot m_e \cdot \beta^{1/2} \approx 2.741525(12) \times 10^{-69} \text{kg} \quad \textbf{(41)}$$

The annual variation in the photon mass Δm_{ph} will be proportional to the ratio between the annual variation of the apparent mass of the universe Δm_u and the apparent mass of the universe m_u :

$$\Delta m_{ph} = m_{ph} \cdot \frac{\Delta m_u}{m_u} \approx -4.0 \times 10^{-79} \frac{\text{kg}}{\text{year}} \quad \textbf{(42)}$$

This value is way too small for us, for the moment, to measure its value.

2.12. Variation of the Planck Mass m_p over Time

Currently, the Planck mass is calculated this way:

$$m_p = \sqrt{\frac{h \cdot c}{2\pi \cdot G}} \quad \textbf{(43)}$$

According to the CODATA 2010, $m_p \approx 2.17651(13) \times 10^{-8}$ kg. This value is not very accurate because of the uncertainty in the universal gravitational constant G.

$$G = \frac{c^2 \cdot r_e \cdot \alpha^{20}}{m_e \cdot \beta} \approx 6.67323036(30) \times 10^{-11} \text{m}^3/(\text{kg} \cdot \text{s}^2) \quad \textbf{(44)}$$

We also know that the mass of the electron m_e may be calculated from the equality between the mass-energy equation and the wave energy as following:

$$m_e \cdot c^2 = \frac{h \cdot c \cdot \alpha}{2 \cdot \pi \cdot r_e} \quad \textbf{(45)}$$

From the equations **(43)**, **(44)** and **(45)**, we may calculate accurately the Planck mass m_p as following:

$$m_p = m_e \cdot \sqrt{\frac{\beta}{\alpha^{21}}} \approx 2.17660867(10) \times 10^{-8} \text{kg} \quad \textbf{(46)}$$

So, this value is 1300 times more accurate than the CODATA 2010 one (see equation **(43)**).

It can be shown that the Planck mass m_p is the geometric mean between the apparent mass of the universe m_u and the mass of photon of lower energy m_{ph}.

$$m_p = \sqrt{m_u \cdot m_{ph}} \tag{47}$$

From the equations **(1)** and **(47)**, we can show that:

$$N = \frac{m_u^2}{m_p^2} = \frac{m_p^2}{m_{ph}^2} \tag{48}$$

The annual variation in the Planck mass Δm_p will be proportional to the ratio between the annual variation of the apparent mass of the universe Δm_u and the mass of the universe m_u:

$$\Delta m_p = m_p \cdot \frac{\Delta m_u}{m_u} \approx -3.2\times 10^{-18}\ \frac{\text{kg}}{\text{year}} \tag{49}$$

The value of the Planck mass may therefore be improved about 400 000 times before we realize that its value changes annually.

2.13. Variation of the Mass of the Electron m_e over Time

At the equation **(46)**, we show that the Planck mass m_p may be described as a function of the mass of the electron m_e. So, the mass of the electron m_e is equal to:

$$m_e = m_p \cdot \sqrt{\frac{\alpha^{21}}{\beta}} \tag{50}$$

Therefore, from the same equation and from the equation **(48)** we get the following equation:

$$N = \frac{m_u^2 \cdot \alpha^{21}}{m_e^2 \cdot \beta} \tag{51}$$

The annual variation in the mass of the electron Δm_e will be proportional to the ratio between the annual change in the mass of the electron Δm_e and the apparent mass of the universe m_u:

$$\Delta m_e = m_e \cdot \frac{\Delta m_u}{m_u} \approx -1{,}3 \times 10^{-40}\ \frac{\text{kg}}{\text{year}} \qquad \textbf{(52)}$$

According to the CODATA 2010 [8], $m_e \approx 9.10938291(40) \times 10^{-31}$ kg. It is therefore possible to improve the measurement of the mass of the electron about 300 times before we realize that its value changes annually.

2.14. Variation of the Planck Length L_p over Time

Because of the Heisenberg uncertainty principle, the Planck length L_p is considered the smallest unit of measurement existing. According to the CODATA 2010, the value of L_p is:

$$L_p = \sqrt{\frac{h \cdot G}{2 \cdot \pi \cdot c^3}} \approx 1.616199(97) \times 10^{-35}\,\text{m} \qquad \textbf{(53)}$$

Let's note here that we omitted to use the equation **(44)** that would have certainly allowed us to use a more accurate value of the universal gravitational constant G and improve the accuracy of the value of the Planck time t_p. However, knowing the equations **(44)** and **(45)**, it is possible to rewrite the equation **(53)** as follows:

$$L_p = r_e \cdot \sqrt{\frac{\alpha^{19}}{\beta}} \approx 1.616125436(53) \times 10^{-35}\,\text{m} \qquad \textbf{(54)}$$

This way of calculating the Planck length is about 18 000 times more accurate than the measured value shown in the CODATA 2010.

Now, let's try to evaluate the annual variation of the Planck length.

The value of the apparent radius of curvature of the luminous universe R_u may be describes as a function of the classical radius of the electron r_e with the following equation:

$$R_u = \frac{r_e}{\beta^{1/2} \cdot \alpha^{19}} \qquad \textbf{(55)}$$

With equations **(4)**, **(54)** and **(55)**, it is possible to show that the constant N can be written in terms of the apparent radius of the luminous universe R_u and the Planck length L_p as follows:

$$N = \frac{R_u^2}{L_p^2} \tag{56}$$

As we have previously shown that N must be constant, the Planck length L_p is obliged to increase over time to keep pace with the change in the apparent radius of curvature of the luminous universe R_u (in the proportion required to keep N constant).

The annual change in the Planck length ΔL_p will be proportional to the ratio between the annual change in the apparent radius of curvature of the luminous universe ΔR_u and the apparent radius of curvature of the luminous universe R_u:

$$\Delta L_p \approx L_p \cdot \left(\frac{\Delta R_u}{R_u}\right) \approx 1.2 \times 10^{-45} \text{ m/year} \tag{57}$$

The Planck length L_p thus increases every year due to the expansion of the universe.

2.15. Variation of the Planck Time t_p over Time

The Planck time t_p is the smallest existing unit of time and it is described by:

$$t_p = \sqrt{\frac{h \cdot G}{2 \cdot \pi \cdot c^5}} \tag{58}$$

According to the CODATA 2010 [8], the Planck time $t_p \approx 5.39106(32) \times 10^{-44}$ s.

Let's note here that we omitted to use equation **(44)** that would have certainly allowed us to use a more accurate value of the universal gravitational constant G and improve the accuracy of the value of the Planck time t_p. However, knowing the equations **(44)** and **(45)**, it is possible to rewrite equation **(58)** as follows:

$$t_p = \frac{r_e}{c} \cdot \sqrt{\frac{\alpha^{19}}{\beta}} \approx 5.39081426(52) \times 10^{-44} \text{s} \tag{59}$$

With the r_e, c and α values coming from the CODATA 2010 [8], the equation **(59)** allows to calculate the value of the Planck time t_p with an accuracy 1200 times better than the equation **(58)**.

Thanks to the equations **(4)**, **(6)**, **(7)** and **(59)**, it is possible to demonstrate that:

$$N = \frac{T_u^2}{t_p^2} = \frac{1}{H_0^2 \cdot t_p^2} \qquad \textbf{(60)}$$

We find that the annual change in the Planck time Δt_p must be proportional to that of the annual change in the apparent age of the universe, that is to say proportional to the inverse of the Hubble constant H_0.

$$\Delta t_p = \frac{t_p \cdot H_0 \cdot 31557600\text{s}}{1\,\text{year}} \approx 4.0 \times 10^{-54}\ \text{s/year} \qquad \textbf{(61)}$$

2.16. Variation of the Planck Frequency f_p over Time

The Planck time t_p is the smallest unit of time that exists. Its inverse therefore leads to the highest frequency that is associated with a particle. This frequency f_p is named " Planck frequency".

The accuracy of the Planck frequency f_p is directly dependent on the accuracy of the Planck time t_p. According CODATA 2010 [8], the Planck time is given by $t_p \approx 5.39106(32) \times 10^{-44}$ s. However, we saw in the equation **(59)** that it is possible to obtain a better estimate of this constant. According to this equation, $t_p \approx 5.3908142(52) \times 10^{-44}$ s. This value leads to the following result:

$$f_p = \frac{1}{2\pi \cdot t_p} = \frac{c}{2 \cdot \pi \cdot r_e} \cdot \sqrt{\frac{\beta}{\alpha^{19}}} \approx 2.9523360(28) \times 10^{42}\,\text{Hz} \qquad \textbf{(62)}$$

Be careful, sometimes, in the literature, this frequency is given in radians/s as follows:

$$f_p = \frac{1}{t_p} \approx 6.9 \times 10^{39}\,\text{radian/s} \qquad \textbf{(63)}$$

The annual variation in the Planck frequency Δf_p , in Hz/year, is given by:

$$\Delta f_p = \frac{1}{2\pi \cdot \text{year}} \cdot \left(\frac{1}{t_p + \Delta t_p} - \frac{1}{t_p} \right) \approx -2.2 \times 10^{32}\,\text{Hz/year} \qquad \textbf{(64)}$$

The annual change in the Planck frequency Δf_p , in radian/(s · year), is given by:

$$\Delta f_p = \frac{1}{\text{year}} \cdot \left(\frac{1}{t_p + \Delta t_p} - \frac{1}{t_p} \right) \approx -1.4 \times 10^{33} \text{radian/(s} \cdot \text{year)} \tag{65}$$

2.17. Variation of the Planck Constant *h* over Time

Currently, the value of Planck constant in the CODATA 2010 is:

$$h \approx 6.62606957(29) \times 10^{-34} \text{J/°K} \tag{66}$$

Let's calculate the annual variation in the Planck constant Δh starting from the equation that makes an equality between the energy contained in the mass of the electron m_e and the wave which is associated with this mass:

$$m_e \cdot c^2 = \frac{h \cdot c \cdot \alpha}{2\pi \cdot r_e} \tag{67}$$

Let's isolate the fine structure constant α:

$$\alpha = \frac{2\pi \cdot r_e \cdot m_e \cdot c}{h} \tag{68}$$

We know that the annual variation in the fine structure constant $\Delta\alpha$ is zero. Therefore, we have:

$$\Delta\alpha = 2\pi \left(\frac{(r_e + \Delta r_e \cdot 1\,\text{year}) \cdot (m_e + \Delta m_e \cdot 1\,\text{year}) \cdot (c + \Delta c \cdot 1\,\text{year})}{(h + \Delta h \cdot 1\,\text{year})} - \frac{r_e \cdot m_e \cdot c}{h} \right) \tag{69}$$

By forcing the annual variation of the fine structure constant to be $\Delta\alpha = 0$ and isolating the annual change in the Planck constant Δh we get:

$$\Delta h = \frac{h \cdot (r_e + \Delta r_e \cdot 1\,\text{year}) \cdot (m_e + \Delta m_e \cdot 1\,\text{year}) \cdot (c + \Delta c \cdot 1\,\text{year})}{r_e \cdot m_e \cdot c} - h \tag{70}$$

Using a few approximations, we get:

$$\Delta h \approx h \cdot \left(\frac{\Delta r_e \cdot 1\,\text{year}}{r_e} \right) \cdot \left(\frac{\Delta m_e \cdot 1\,\text{year}}{m_e} \right) \cdot \left(\frac{\Delta c \cdot 1\,\text{year}}{c} \right) \tag{71}$$

Then, if we evaluate the value of the variation in Planck constant Δh, we get:

$$\Delta h \approx -5.3 \times 10^{-64} \text{J} \cdot \text{s/year} \tag{72}$$

This variation is so small compared to the uncertainty of this constant that we could consider the Planck constant *h* as being really constant. This assertion makes possible to maintain the principle of conservation of energy through the universe over time.

2.18. Variation of the Universal Gravitational Constant *G* over Time

Thanks to our previous work, we know that the universal gravitational constant *G* is given by the following equation:

$$G = \frac{c^2 \cdot r_e \cdot \alpha^{20}}{m_e \cdot \beta} \approx 6.67323036(30) \times 10^{-11}\ \mathrm{m^3/(kg \cdot s^2)} \tag{73}$$

As we have already calculated the variations related to each of the physics constants involved, we are able to calculate the annual variation in the gravitational constant ΔG:

$$\Delta G = \frac{\alpha^{20}}{\beta \cdot 1\,\mathrm{year}} \cdot \left(\frac{(c + \Delta c \cdot 1\,\mathrm{year})^2 \cdot \left(r_e + \Delta r_e \cdot 1\,\mathrm{year}\right)}{\left(m_e + \Delta m_e \cdot 1\,\mathrm{year}\right)} - \frac{c^2 \cdot r_e}{m_e} \right) \tag{74}$$

$$\Delta G \approx 2.5 \times 10^{-20}\,\mathrm{m^3/(kg \cdot s^2 \cdot year)} \tag{75}$$

We find that the gravitational constant *G* increases over time. The accuracy of this constant may be improved by a factor of about 325 000 with respect to the value shown in the CODATA 2010 [8] which is $G \approx 6.67384(80) \times 10^{-11}\ \mathrm{m^3/(kg \cdot s^2)}$. It could be improved by a factor of about 120 if we use equation **(73)**.

We immediately note that, contrary to what some might think, the variation over time of the gravitational constant *G* can not explain the phenomenon of the Pioneer effect. Indeed, once we have also taken into account the variation of masses and distances over time (due to the expansion of the universe), this effect is significantly less than the Pioneer effect. In our opinion, only the acceleration of light over time succeeds to fully explain the Pioneer effect.

2.19. Variation of the Rydberg Constant R_∞ over Time

The Rydberg constant may be written this way:

$$R_\infty = \frac{\alpha^3}{4\cdot\pi\cdot r_e} \approx 10973731.568539(55)\ \mathrm{m}^{-1} \tag{76}$$

As the fine structure constant is constant over time, the annual variation in the Rydberg constant ΔR_∞ is proportional to the inverse of the annual variation of the classical radius of the electron Δr_e.

$$\Delta R_\infty \approx \frac{\alpha^3}{4\cdot\pi\cdot\mathrm{year}}\cdot\left(\frac{1}{r_e+\Delta r_e\cdot 1\,\mathrm{year}}-\frac{1}{r_e}\right) \approx -8.1\times 10^{-4}\ \mathrm{m}^{-1}/\mathrm{year} \tag{77}$$

We conclude that the uncertainty of the Rydberg constant R_∞ is slightly underestimated by a factor of about 15, compared to the value given in CODATA 2010. Indeed, it may be more accurate its annual variation. The authors of the experiment that led to the assessment of the Rydberg constant are probably not repeated enough times that their experience over time to realize that it varied slightly. In the next printing of updated values CODATA, the value of the Rydberg constant R_∞ therefore is likely to change downward.

It would be interesting to know the history of the evaluation of the Rydberg constant R_∞ over time. Its evolution may help to highlight, indirectly, the expansion of the universe and, by extension, validate a part of our model of the universe. Unfortunately, if we rely on the latest versions of CODATA available or we go back too far in time, the obtained values were not precise enough to test our hypothesis, because the error margin is too large. The next future values that will be measured in the coming years will probably help to verify our hypothesis.

2.20. Variation of the Charge of the electron q_e over Time

Einstein showed that the energy contained in the mass of an electron is:

$$E_e = m_e\cdot c^2 \tag{78}$$

The classical radius of an electron r_e comes from the classical electrostatics. In fact, it does not really have that radius because the electron is point-like. However, it is generally accepted that all the energy E_e of an electron is entirely contained within that radius. Therefore, if we take the charge of an electron q_e and as we near the center of the charge q_e to another electron, we will perform a work. If we bring the charge to a distance r_e from the center of the electron, the energy in this charge will be entirely due to its electrostatic energy. We will then have the following equation:

$$E_e = F \cdot r_e = \frac{q_e^2}{4 \cdot \pi \cdot r_e^2} \cdot r_e = \frac{q_e^2}{4 \cdot \pi \cdot r_e} \qquad \textbf{(79)}$$

Using the equations **(26)**, **(78)** and **(79)** we obtain an equation that gives the charge of an electron as a function of the mass m_e and classical radius r_e.

$$q_e = -\sqrt{\frac{4 \cdot \pi \cdot m_e \cdot r_e}{\mu_0}} \approx -1.602176565(35) \times 10^{-19}\ \mathrm{C} \qquad \textbf{(80)}$$

From a mathematical point of view, we must put ± in front of the square root. Of course, the charge of the electron q_e is negative. However, the charge of the positron q_e + is positive.

The annual variation of the charge of the electron Δq_e will therefore be:

$$\Delta q_e = \frac{-\sqrt{\frac{4\pi}{\mu_0}}}{1\,\mathrm{year}} \cdot \left(\sqrt{\left(m_e + \Delta r_e \cdot 1\,\mathrm{year}\right) \cdot \left(r_e + \Delta r_e \cdot 1\,\mathrm{year}\right)} - \sqrt{m_e \cdot r_e} \right) \qquad \textbf{(81)}$$

$$\Delta q_e \approx 5.9 \times 10^{-30}\ \mathrm{C/year} \qquad \textbf{(82)}$$

Similarly, the annual variation in the charge of the positron q_e+ becomes:

$$\Delta q_e \approx -5.9 \times 10^{-30}\ \mathrm{C/year} \qquad \textbf{(83)}$$

We conclude that it is possible to improve, by a factor of about 4 200, the accuracy on the measurement of the charge of the electron q_e and of the positron q_e + currently known in the CODATA 2010 [8]. Then, we would see an annual variation in the least significant digits.

2.21. Variation of the Von Klitzing Constant R_k over Time

Discovered in 1980 by the physicist Klauss Von Klitzing, the Von Klitzing constant, in quantum physics, allows to calculate with great precision the value of electrical resistance. The Klitzing resistance R_k is related to the impedance Z_0 of the vacuum.

$$R_k = \frac{h}{q_e^2} = \frac{Z_0}{2 \cdot \alpha} = \frac{\mu_0 \cdot c}{2 \cdot \alpha} \approx 25812.8074434(84)\Omega \tag{84}$$

Let's use the last equation that depends on the speed of light c. The speed of light is the only none constant parameter of this equation, since we have previously shown that μ_0 and α are constant.

The annual variation in the Klitzing resistance ΔR_k becomes:

$$\Delta R_k = \frac{\mu_0 \cdot \Delta c}{2 \cdot \alpha} \approx 1.9 \times 10^{-6} \ \Omega/\text{year} \tag{85}$$

We find that this constant is known fairly accurately because the annual variation is only 4 times smaller than the uncertainty of R_k. If we reduce the uncertainty by a factor 4, then, it would be possible to note an annual change of this constant.

2.22. Variation of the Josephson Effect Constant k_j over Time

The Josephson effect is manifested by the appearance of a current between two superconductors separated by a layer made of a non-superconducting metal or an insulating material [23]. There is a Josephson effect in direct current and in alternative current. According to the CODATA 2010 [8], the value of the Josephson constant k_j is the following:

$$k_j = \frac{2 \cdot q_e}{h} \approx -483597.870(11) \times 10^9 \ \text{Hz/Volt} \tag{86}$$

We find that the Josephson constant is a function of the Planck constant and of the electric charge q_e which both vary over time. The annual variation of the Josephson constant Δk_j is given by:

$$\Delta k_j = 2\left(\frac{q_e + \Delta q_e \cdot 1\,\text{year}}{h + \Delta h \cdot 1\,\text{year}} - \frac{q_e}{h}\right) \tag{87}$$

$$\Delta k_j \approx -1.8 \times 10^4 \ \text{Hz/(Volt} \cdot \text{year)} \tag{88}$$

It would therefore be possible to improve this measure by a factor about 600 before finding an annual variation of this physics constant.

2.23. Variation of the Average Temperature of the Cosmic Microwave Background of the Universe T_u over Time

To maintain the principle of conservation of energy, the total energy E_u contained in the universe is constant, the average temperature T of the cosmic microwave background (CMB) is decreasing. For proof, the average temperature of the CMB is now around 2.7 K° when the universe had inevitably a higher temperature in its beginning. In previous work, we calculated the average temperature T of the CMB:

$$T = \frac{m_e \cdot c^2}{k_b} \cdot \left(\frac{15 \cdot \beta^6 \cdot \alpha^{17}}{\pi^3} \right)^{1/4} \approx 2.736795(3)\ °K \tag{89}$$

According to the CODATA 2010 [8] :

- Boltzmann Constant $k_B \approx 1.3806488(13) \times 10^{-23}$ J/°K

If the big bang theory is correct, when the universe began its expansion from a singularity point, all matter was confined in a very small point. The pressure was its maximum in an extremely small volume. We may therefore assume that the temperature was also at its maximum. This is what we call the Planck temperature which corresponds to the top limit for the temperature scale:

$$T_p = \frac{m_p \cdot c^2}{k_b} = \frac{m_e \cdot c^2}{k_b} \cdot \sqrt{\frac{\beta}{\alpha^{21}}} \approx 1.4169 \times 10^{32}\ °K \tag{90}$$

Starting from the Planck temperature T_p, the temperature began to decrease over time to asymptotically approach the temperature of absolute zero. As long as the universe will continue to expand, the average temperature of the cosmic microwave background will continue to drop slowly. This type of decay is exponential decreasing type. If we know that the starting temperature is T_p, and the actual temperature is $T \approx 2.736795$ °K after a time equal to the apparent age of the universe $T_u = 1/H_0$, we are able to establish an equation that gives the temperature as a function of time.

$$T_p \cdot e^{-T_u/\tau} = T_p \cdot e^{-1/(H_0 \cdot \tau)} = T \tag{91}$$

Let's isolate the time constant τ used to calculate the decay of the average temperature T of the cosmic microwave background:

$$\tau = \frac{-1}{H_0 \cdot \ln\left(\dfrac{T}{T_p}\right)} = \frac{-4 \cdot r_e}{c \cdot \alpha^{19} \cdot \beta^{1/2} \cdot \ln\left(\dfrac{15 \cdot \beta^4 \cdot \alpha^{59}}{\pi^3}\right)} \tag{92}$$

$$\tau \approx 5.861036698(37) \times 10^{15} \text{ s} \approx 186 \text{ million years} \tag{93}$$

We are now able to calculate the annual variation in the average temperature of the cosmic microwave background ΔT :

$$\Delta T = \frac{T_p \cdot e^{-\frac{1}{\tau}} \cdot \left(e^{\left(\frac{1}{H_0} - 31557600\text{s}\right)} - 1\right)}{\text{year}} \approx -1.5 \times 10^{-8}\ {}^\circ\text{K/year} \tag{94}$$

We conclude that it is possible to improve the precision of T in equation **(89)** by a factor of about 200. After that, we would see an annual variation that would affect the accuracy of the measurements.

2.24. Variation of the Boltzmann Constant k_b over Time

According to the CODATA 2010 [8], the Boltzmann constant is given by $k_b \approx 1.3806488(13) \times 10^{-23}$ J/°K.

To maintain the principle of conservation of the energy, the energy E given by the following equation must be constant:

$$E = k_b \cdot T = \text{Constant} \tag{95}$$

When the average temperature of the cosmic microwave background decreases over time, the Boltzmann constant k_B is obliged to increase to conserve the energy E constant.

Therefore, to ensure that equality of equation **(95)** is maintained, it is necessary that the annual variation in the Boltzmann constant Δk_B increases in the same proportion as the annual variation in the average temperature of the cosmic microwave background:

$$\left(k_b + \Delta k_b \cdot 1\,\text{year}\right) \cdot \left(T + \Delta T \cdot 1\,\text{year}\right) = k_b \cdot T \tag{96}$$

From this equation, we get:

$$\Delta k_b = \frac{k_b}{1\,\text{year}} \cdot \left(\frac{T}{(T + \Delta T \cdot 1\,\text{year})} - 1 \right) \approx 7.4 \times 10^{-32} \text{J/(°K} \cdot year) \quad \textbf{(97)}$$

We note that it would be theoretically possible to improve the accuracy of the Boltzmann constant by a factor of about 70. After that, we would see an annual variation that would affect the accuracy of the measurements.

2.25. Variation of the Universal Gas Constant $\Re$ over Time

The universal gas constant $\Re$ is used in the ideal gas law.

$$P \cdot V = n \cdot \Re \cdot T \quad \textbf{(98)}$$

In this equation, P is the pressure inside a chamber of volume V. The temperature of the chamber is T and the number of gas molecules is n. Constant $\Re$ acts as a proportionality constant which adapts the measurement units:

The universal gas constant $\Re$ comes itself from the product of the Boltzmann constant k_b by the Avogadro number N_A:

$$\Re = k_b \cdot N_A \approx 8.3144621(75)\,\text{J/(mol} \cdot \text{°K)} \quad \textbf{(99)}$$

According CODATA 2010 [8], the Avogadro number is given by $N_A \approx 6.02214129(27) \times 10^{23}$ mol^{-1}. This number can not change over time as it represents the number of atoms per mol.

$$N_A = \text{constant} \approx 6.02214129(27) \times 10^{23}\,\text{mol}^{-1} \quad \textbf{(100)}$$

The annual change in the universal gas constant $\Delta\Re$ is given by:

$$\Delta\Re = \Delta k_b \cdot N_A \approx 4.5 \times 10^{-8}\,\text{J/(year} \cdot \text{mol} \cdot \text{°K)} \quad \textbf{(101)}$$

We find that it would theoretically be possible to improve the accuracy of the universal gas constant $\Re$ by a factor of about 165. After that, we would see an annual variation that would affect the accuracy of measurements.

2.26. Variation of the Stefan-Boltzmann Constant σ over Time

The Stefan-Boltzmann constant is given by the following equation:

$$\sigma = \frac{2 \cdot \pi^5 \cdot k_b^4}{15 \cdot h^3 \cdot c^2} \quad \textbf{(102)}$$

According to the CODATA 2010 [8], the Stefan-Boltzmann constant is given by

$\sigma \approx 5.670373(21) \times 10^{-8}$ W/(m²·°K⁴).

The annual variation of the Stefan-Boltzmann constant $\Delta\sigma$ is therefore:

$$\Delta\sigma = \frac{2 \cdot \pi^5}{15 \cdot 1\,\text{year}} \cdot \left[\frac{\left(k_b + \Delta k_b \cdot 1\,\text{year}\right)^4}{\left(h + \Delta h \cdot 1\,\text{year}\right)^3 \cdot \left(c + \Delta c \cdot 1\,\text{year}\right)^2} - \frac{k_b^4}{h^3 \cdot c^2} \right] \quad \textbf{(103)}$$

$$\Delta\sigma \approx 1.2 \times 10^{-15}\,\text{W/(m}^2 \cdot {}^\circ\text{K}^4 \cdot \text{year)} \quad \textbf{(104)}$$

We note that it would theoretically be possible to improve the accuracy of the Stefan-Boltzmann constant by a factor of about 175. After that, we would see an annual variation that would affect the accuracy of measurements.

2.27. Variation of the Ratio Between the Mass of the Electron m_e and the Mass of the Proton m_{pr}

As we mentioned earlier, ratios between numerators and denominators that have the same units produce results without units that do not vary over time. Thus, the ratio between the mass of the electron and the proton is interesting because it is constant over time.

The mass of the proton is $m_{pr} \approx 1.672621777(74) \times 10^{-27}$ kg is about 1836 times larger than that of the electron that is $m_e \approx 9.10938291(40) \times 10^{-31}$ kg. It is therefore easier to measure. However, despite this, the ratio m_e/m_{pr} is known with an accuracy better than each of the individual terms used to make the ratio (according the value written in the CODATA 2010 [8], $m_e/m_{pr} \approx 5.4461702178(22) \times 10^{-4}$) . This is normal, because the ratio does not change over time as the annual change in the numerator (in percentage) is compensated by the annual change in the denominator. So the variations cancel and the resulting ratio is constant.

$$\frac{m_e}{m_{pr}} = \text{constant} \quad \textbf{(105)}$$

3. CONCLUSION

As Dirac had suspected [1], several physics "constants" vary over the years due to the expansion of the universe. For now, these variations are less than or up to the limit of the experimental measurement errors. Therefore, with the techniques and technologies used, it is very difficult to observe these annual variations. It is not our intention to blame anyone giving wrongly the title of "constant" to certain physics parameters. We must understand that for the time scale of a human, some variations become imperceptible. It is therefore perfectly legitimate and even useful to consider certain parameters as being constant. Especially if we are able to describe some phenomena using equations which use these parameters.

Like what was mentioned in our introduction, we recall here that our work was to calculate variations in various physics constants **if we accept the fact that light accelerates over time.** Indeed, as long as the constants are defined according to the speed of light c, it will not be possible to detect variations of these constant over time. Considering the speed of light sets, we will discover phenomena such as the Pioneer effect, which will be difficult to explain.

In order to provide us the most reliable and reproducible standards possible over time, it may seem useful and necessary (for now) to assume that the speed of light in vacuum c is constant. But this trick should not blind us on the fact that it is also essential to take account of the change in the speed of light over time to explain the phenomena around us. The physics phenomena are described by laws that are, among others, what they are due to the fact that our universe is expanding.

Understanding the different variation mechanism in physics parameters is critical to know the allowable limits margins of error in measurements. Indeed, it may seem trivial to write a number in a table which will anyway be readjusted in the following year. Usually it is better to know the part of the number that will be stable over a period of time considered significant to our time scale. If this can not be done, then it is better to create an equation that gives the state of the physical parameter as a function of time so to keep a certain validity of measured significant digits.

In this paper, we first try to identify the actual physics constants, those that were independent of the expansion mechanism of the universe. When the constants came from geometrical results (such as β and k), their constancy was indisputable. When they were the result of ratios of two constants having the same units (such as N and α), annual changes automatically cancel and create real physics constants.

From these real constants and using different known equations, it was possible to systematically establish the annual variations in other parameters commonly used in physics (which we mistakenly call "physics constants").

Our study allows to know the different variation mechanisms involved. We summarize the results of these calculations in a table in the appendix highlighting the annual variations of the different physics parameters.

Perhaps this document could be used to establish which parameters of the universe should be encouraged to become standards in the international measurement system. As we have seen, some vary less than others.

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SUMMARY TABLE OF THE ANNUAL VARIATIONS OF THE DIFFERENT PHYSICS PARAMETERS DUE TO THE EXPANSION OF THE UNIVERSE

Description of the constant	Value and (accuracy)	Source	Equation	Variation /year
1- Fine structure constant	$\alpha \approx 7{,}2973525698(24) \times 10^{-3}$	[8]	measured	$\Delta\alpha$ = Constant
2- Maximum number of photons of $2\pi{\cdot}R_u$ wavelength	$N \approx 6{,}30341951(12) \times 10^{121}$	[7]	**(4)**	ΔN = Constant
3- Ratio speed of material universe / speed of c	$\beta \approx 0{,}7639320225$	[3]	**(8)**	$\Delta\beta$ = Constant
4- Speed limit of light when $R_u \to \infty$	$k \approx 6{,}17024151 \times 10^{8}$ m/s	[3]	**(9)**	Δk = Constant
5- Vacuum permeability	$\mu_0 \approx 4{\cdot}\pi \times 10^{-7}$ N/A^2	[8]	defined	$\Delta\mu_0$ = Constant
6- Avogadro number	$N_A \approx 6{,}02214129(27) \times 10^{23}$ mol^{-1}	[8]	measured	ΔN_A = Constant
7- Vacuum permittivity	$\varepsilon_0 \approx 8{,}854187817 \times 10^{-12}$ F/m	[8]	measured	$\Delta\varepsilon_0 \approx -1{,}3\times10^{-21}$ s^4/(kg·m^3·year)
8- Ratio between the mass of the electron and the proton	$m_e / m_{pr} \approx 5{,}4461702178(22) \times 10^{-4}$	[8]	measured	$\Delta m_e / m_{pr}$ = Constant
9- Planck constant	$h \approx 6{,}62606957(29) \times 10^{-34}$ J·s	[8]	measured	$\Delta h \approx -5.3\times10^{-64}$ J·s/year ≈ Constant
10- Speed of light in vacuum	$c \approx 299792458$ m/s	[8]	measured	$\Delta c \approx 0.022$ m/(s·year)
11- Vacuum impedance	$Z_0 \approx 376{,}730313\ \Omega$	[8]	**(31)**	$\Delta Z_0 \approx 2.8 \times 10^{-8}$ Ω/year
12- Universal gravitational constant	$G \approx 6{,}67323036(30) \times 10^{-11}$	[7]	**(44)**	$\Delta G \approx 2.5 \times 10^{-20}$ m^3/(kg·s^2·year)
13- Hubble constant	$H_0 \approx 72{,}09548632(46)$ km/(s·MParsec)	[7]	**(7)**	$\Delta H_0 \approx -5.3 \times 10^{-9}$ km/(s·MParsec·year)
14- Planck frequency	$f_p \approx 2{,}9523360(28) \times 10^{42}$ Hz	This document	**(62)**	$\Delta f_p \approx -2.2 \times 10^{32}$ Hz/year
15- Planck time	$t_p \approx 5{,}3908142(52) \times 10^{-44}$ s	This document	**(59)**	$\Delta t_p \approx 4.0 \times 10^{-54}$ s/year
16- Planck length	$l_p \approx 1{,}616125436(53) \times 10^{-35}$ m	This document	**(54)**	$\Delta l_p \approx 1.2 \times 10^{-45}$ m/year
17- Planck mass	$m_p \approx 2{,}17660867(10) \times 10^{-8}$ kg	This document	**(46)**	$\Delta m_p \approx -3.2 \times 10^{-18}$ kg/year
18- Apparent mass of the universe	$m_u \approx 1{,}72809823(8) \times 10^{53}$ kg	[7]	**(36)**	$\Delta m_u \approx -2.5 \times 10^{43}$ kg/year
19- Apparent radius of curvature of the luminous	$R_u \approx 1{,}2831078806(68)\times10^{26}$ m	[9]	**(14)**	$\Delta R_u \approx 9.5 \times 10^{15}$ m/year
20- Apparent radius of curvature of the material universe	$r_u \approx 9{,}802071983(52) \times 10^{25}$ m	[9]	**(17)**	$\Delta r_u \approx 7.2 \times 10^{15}$ m/year
21- Mass associated to the photon $2\pi{\cdot}R_u$ wavelength	$m_{ph} \approx 2{,}741525(12) \times 10^{-69}$ kg	This document	**(41)**	$\Delta m_{ph} \approx -4.0 \times 10^{-79}$ kg/year
22- Masse of the electron	$m_e \approx 9{,}10938291(40) \times 10^{-31}$ kg	[8]	measured	$\Delta m_e \approx -1.3 \times 10^{-40}$ kg/year
23- Classical radius of the electron	$r_e \approx 2{,}817940326\ 7(27) \times 10^{-15}$ m	[8]	measured	$\Delta r_e \approx 2.1 \times 10^{-25}$ m/year
24- Charge of the electron	$q_e \approx -1{,}602176565(35) \times 10^{-19}\ C$	[8]	measured	$\Delta q_e \approx +5.9 \times 10^{-30}$ C/year
25- Charge of the positron	$q_{e+} \approx +1{,}602176565(35) \times 10^{-19}\ C$	[8]	measured	$\Delta q_{e+} \approx -5.9 \times 10^{-30}$ C/year
26- Rydberg constant	$R_\infty \approx 10973731{,}568539(55)$ m^{-1}	[8]	measured	$\Delta R_\infty \approx -8.0 \times 10^{-4}$ m-1/year
27- Average temperature of the CMB	$T \approx 2{,}736795(3)$ °K	This document	**(89)**	$\Delta T \approx -1.5 \times$ 10-8 °K/year
28- Boltzman constant	$k_b \approx 1{,}3806488(13) \times 10^{-23}$ J/°K	[8]	measured	$\Delta k_b \approx 7.4 \times$ 10-32 J/(°K·year)
29- Stefan-Boltzman constant	$\sigma \approx 5{,}670373(21) \times 10^{-8}$ W/(m^2·°K)	[8]	**(102)**	$\Delta\sigma \approx 1.2 \times 10^{-15}$ W/(m^2·°K·year)
30- Von Klitzing constant	$R_k \approx 25812{,}8074434(84)\ \Omega$	[8]	**(84)**	$\Delta R_k \approx 1.9 \times 10^{-6}$ Ω/year
31- Josephson effect constant	$k_j \approx -483597{,}870(11) \times 10^{9}$ Hz/Volt	[8]	**(86)**	$\Delta k_j \approx -1.8 \times 10^{4}$ Hz/(Volt·year)
32- Universal gas constant	$\mathfrak{R} \approx 8{,}3144621(75)$ J/(mol·°K)	[8]	measured	$\Delta\mathfrak{R} \approx 4.5 \times 10^{-8}$ J/(mol·°K·year)