# Links between the Beckenstein-Hawking Temperature and the Hagedorn Temperature

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*A black hole is a mass that is so dense that it influences the index of refraction of the vacuum around it such a way to create a horizon inside through which no light can escape from.*

*Some times after Jacob Beckenstein has predicted that a black hole might have a temperature and an entropy different from zero, Stephen Hawking predicted, in 1974, that it should produce a radiation that he named "Hawking radiation"* [1]*. This radiation is the basis of the evaporation of black holes over the time. Because of this radiation, a black hole has a surface temperature, called Beckenstein-Hawking temperature (sometimes over time simply named Hawking temperature), which is inversely proportional to its mass. For a massive black hole, this temperature is extremely low.*

*At the other end of the temperature scale, it exists, for a given mass, a temperature so high that all matter is transformed into pure energy (in photons). This theoretical temperature limit, discovered in 1963, is called the Hagedorn temperature* [2,3]*. Contrarily to the Beckenstein-Hawking temperature, the Hagedorn temperature is proportional to the mass of the object in cause.*

*Thanks to the hypothesis on large numbers of Dirac and to some previous works which fine-tune this hypothesis, we established certain relationships between the Beckenstein-Hawking temperature and the Hagedorn temperature. Thanks to the constant $\beta$ (which represents the ratio between the expansion speed of the material universe and the speed of light) coming from works on the acceleration of light over time* [4]*, we even established certain links with the characteristics of the electron.*

**KEY WORDS:** Temperature, Beckenstein, Hawking, Hagedorn, Planck, universe, electron

## 1. INTRODUCTION

Thanks to the Dirac's large numbers hypothesis, certain astrophysicists, like Sidharth [2], noticed that there seems to be a link between the Beckenstein-Hawking temperature and the Hagedorn one. Because of the imprecision of the great numbers that we talk about, they unfortunately could not make the equations equal. Thanks to the fine tuning that we brought to the nature of these great numbers (which, in fact, all come from only one number that we baptized *N*), we can establish exact relations between the Beckenstein-Hawking temperature and the Hagedorn one. We can even make certain links with the classical characteristics of the electron.

As the reader will notice, we found a multitude of equations which could all be demonstrated with the help of simple basic equations. However, even if they are relatively simple to do, there are a lot of them. Our goal is not to demonstrate them since it would be a useless waste of time. We will only show the main equations that can be used to do the demonstrations and then, we will list the different equations that we found. The demonstrations are left to the reader. However, before making the listing, we made sure that all the links can be demonstrated as being exact provided the equations **(1)** to **(15)** are considered exact.

## 2. SOME USEFUL PARAMETER VALUES

### 2.1. Theoretical Hubble Constant $H_0$ from Previous Works

In previous works, we showed that the Hubble constant $H_0$ could be expressed by an equation whose accuracy depended on the speed of light $c$, the fine structure constant $\alpha$ and classical radius of the electron $r_e$.

$$H_0 = \frac{c \cdot \alpha^{19} \cdot \beta^{1/2}}{r_e} \quad \textbf{(1)}$$

$$H_0 \approx 72.09548632 \pm 0.00000046 \mathrm{km}/\left(s \cdot MPar\mathrm{sec}\right) \quad \textbf{(2)}$$

According to CODATA 2010 [7]:

- Actual speed of light in vacuum $c \approx 299792458$ m/s
- Fine structure constant $\alpha \approx 7.2973525698 \pm 0.0000000024 \times 10^{-3}$
- Classical radius of the electron $r_e \approx 2.8179403267 \pm 0.0000000027 \times 10^{-15}$ m

The value of $\beta$ is a pure number. It shows the ratio between the expansion speed of the material universe and the speed of light in vacuum $c$ [4]:

$$\beta = 3 - \sqrt{5} \approx 0.764 \quad \textbf{(3)}$$

We will see that this constant is very important in the evaluation of many known constants in physics (such $H_0$ and $G$) and in many links that we will establish between the Beckenstein-Hawking temperature $T_B$ and the Hagedorn temperature $T_H$.

The value of the Hubble constant obtained in **(1)** is compatible with the one found by Xiaofeng Wang and his team [12] who obtained the following measurement value: $H_0$ = 72.1 ±0.9 km/(s·MParsec). So, we will use our equation in the present paper to describe the Hubble constant $H_0$.

### 2.2. Theoretical Universal Gravitational Constant *G* from Previous Works

In previous works, we have already shown that the universal gravitational constant $G$ can be described with great precision by using the following equation which mainly depends on the fine structure constant $\alpha$, on the classical radius of the electron $r_e$ , on its mass $m_e$ and on the speed of light in vacuum $c$ :

$$G = \frac{c^2 \cdot r_e \cdot \alpha^{20}}{m_e \cdot \beta} \approx 6.67323036 \pm 0.0000003 \times 10^{-11}\ \mathrm{m}^3 / \left(kg \cdot s^2\right) \qquad \textbf{(4)}$$

According to the CODATA 2010 [7]:

- Universal gravitational constant $G \approx 6.67384 \pm 0.00080 \times 10^{-11}$ $\mathrm{m}^3/(\mathrm{kg}\cdot\mathrm{s}^2)$
- Mass at rest of the electron $m_e \approx 9.10938291 \pm 0.00000040 \times 10^{-31}$ kg.

Since the value obtained in **(4)** is compatible with the CODATA 2010 error margin, we will use this value in the present article to describe the universal gravitational constant $G$.

### 2.3. Number *N* of Dirac

In 1974, Dirac made the hypothesis that certain large numbers keep coming back when we evaluate the ratio of certain numbers [8]. We established that all these numbers came, in fact, from only one large number $N$ which is in the order of $10^{121}$ [9]. By applying different rational exponents to $N$ (e.g. ½, ⅓, etc.), we get numbers that have the same order of magnitude as all the great numbers of the original Dirac hypothesis.

In previous works [9], we have shown that if we associate a mass $m_{ph}$ to the photons that have a wavelength equal to the apparent circumference of the universe ($2\cdot\pi\cdot R_u$), the number $N$ therefore corresponds to the highest number of photons that may exist in our universe which has an apparent mass equal to $m_u$.

The mass associated to a photon of wavelength equal to $2\cdot\pi\cdot R_u$ (the apparent circumference of the universe) is given by $m_{ph}$ [13]:

$$m_{ph} = \frac{h}{2 \cdot \pi \cdot R_u \cdot c} = \frac{h \cdot H_0}{2 \cdot \pi \cdot c^2} \approx 2.74 \times 10^{-69} \mathrm{kg} \quad \textbf{(5)}$$

The apparent mass of the universe is given by the following equation [14,15]:

$$m_u = \frac{c^3}{G \cdot H_0} \approx 1.73 \times 10^{53} \mathrm{kg} \quad \textbf{(6)}$$

In other previous works that we have already published [10], we have made the hypothesis that $N$ was intimately related to the fine structure constant $\alpha$.

$$N = \frac{m_u}{m_{ph}} = \frac{1}{\alpha^{57}} \approx 6.30 \times 10^{121} \quad \textbf{(7)}$$

This conjecture allowed finding the equations **(1)** and **(4)**. For now, we cannot make a formal demonstration of the equations **(1)**, **(4)** and **(7)** from known theories. However, the veracity of these is indirectly demonstrated by the fact that, with the latter, we can calculate precisely [11] the mass of the electron $m_e$ and get exactly the same mass value of the electron as that of the CODATA 2010 [7]. Considering that the mass of the electron $m_e$ is described with more than eight significant digits and since we get the same ones [11], we consider that this situation may not statistically be random.

## 2.4. Relation between the Apparent Radius of Curvature of the Universe $R_u$ and the Classical Radius of the Electron $r_e$

The apparent radius of curvature of the universe (sometime called the Hubble radius [16], radius of the universe [18], radius of curvature of spacetime [19]) is normally given by the following equation:

$$R_u = \frac{c}{H_0} \approx 1.28 \times 10^{26} \mathrm{m} \quad \textbf{(8)}$$

However, thanks to the value of the Hubble constant $H_0$ from the equation **(1)**, we can determine precisely the value of the apparent radius of curvature of the universe $R_u$:

$$R_u = \frac{r_e}{\beta^{1/2} \cdot \alpha^{19}} = \frac{r_e \cdot N}{\beta^{1/2}} \approx 1.2831078806 \pm 0.0000000081 \times 10^{26} \mathrm{m} \quad \textbf{(9)}$$

Many other methods are described in a document that we published previously [6].

The equation **(9)** will allow us to find certain equations showing links between the Beckenstein-Hawking temperature $T_B$ , the Hagedorn temperature $T_H$ and the classical radius of the electron $r_e$ .

### 2.5. Beckenstein-Hawking Temperature $T_B$

Following the Beckenstein work, Stephen Hawking discovered in 1974 that, as an ideal black body, black holes emit a radiation that is the cause of the evaporation of these over time [1]. He discovered the theoretical link (function $T_B(m)$) that allowed to calculate the surface temperature $T_B$ (at the horizon) of black holes as a function of their mass $m$ [2].

$$T_B(m) = \frac{\hbar \cdot c^3}{8 \cdot \pi \cdot k_b \cdot G \cdot m} = \frac{h \cdot c^3}{16 \cdot \pi^2 \cdot k_b \cdot G \cdot m} \tag{10}$$

We realize that the temperature $T_B$ is even lower as much as that of the mass of the considered black hole is high.

### 2.6. Hagedorn Temperature $T_H$

Einstein showed that we could convert a given quantity of matter of mass $m$ into pure energy $E$ (in photons) by using the following equation [5]:

$$E = m \cdot c^2 \tag{11}$$

The thermal energy $E_t$ of a body that is at a temperature $T$ is given by the following equation:

$$E_t = k_b \cdot T \tag{12}$$

By making the equations **(11)** and **(12)** equal and by supposing that the thermal energy is so high that all the mass $m$ gets transformed entirely into photons, we obtain the following function $T_H(m)$ which gives the Hagedorn temperature $T_H$ as a function of the mass $m$:

$$T_H(m) = \frac{m \cdot c^2}{k_b} \tag{13}$$

It is possible, among other things, to find this equation in different works of Sidharth [2] and of Sivaram [3].

We realize that contrary to the Beckenstein-Hawking temperature, the Hagedorn temperature $T_H$ is as high as the considered mass is high.

### 2.7. Planck Temperature $T_P$

In the particular case where the mass $m$ is the Planck mass $m_p$ in the equation **(13)**, the Hagedorn temperature $T_H$ of the equation **(13)** becomes equal to the Planck temperature $T_P$ (see equation **(14)**). Since the Planck mass $m_p$ corresponds to the highest level of energy that a particle may reach, the Planck temperature corresponds to the highest temperature that may exist. A bit like the absolute zero, the Planck temperature $T_P$ corresponds to the highest limit of the temperature scale.

$$T_P = \frac{m_p \cdot c^2}{k_b} \approx 1.4169 \times 10^{32} \text{ °K} \tag{14}$$

Here, the Planck mass $m_p$ corresponds to:

$$m_p = \sqrt{\frac{h \cdot c}{2 \cdot \pi \cdot G}} \tag{15}$$

## 3. DEVELOPEMENT

### 3.1. Relations between the Beckenstein-Hawking and the Hagedorn Temperatures

Without trying to make the mathematical development of each one, we present here some interesting equalities. It will be up to the reader to demonstrate them from the equations shown previously.

<u>Despite the equality of certain equations</u>, we want to inform the reader that certain equalities are purely mathematical since, in fact, the value of certain temperatures would surpass the Planck temperature $T_p$, which is physically impossible. For example, the following combinations are impossible: $T_H(m_u)$, $T_B(m_{ph})$, $T_H(m_e)$.

So, this clarification being done, here are the different equations:

$$N = \frac{8 \cdot \pi \cdot T_B(m_{ph})}{T_H(m_{ph})} = \frac{T_H(m_u)}{8 \cdot \pi \cdot T_B(m_u)} \tag{16}$$

$$N=\left(\frac{T_B(m_{ph})}{T_B(m_p)}\right)=\left(\frac{T_B(m_p)}{T_B(m_u)}\right)^2=\left(\frac{T_H(m_p)}{T_H(m_{ph})}\right)^2=\left(\frac{T_H(m_u)}{T_H(m_p)}\right)^2 \quad \textbf{(17)}$$

$$N=\left(\frac{8\cdot\pi\cdot T_B(m_{ph})}{T_H(m_p)}\right)^2=\left(\frac{8\cdot\pi\cdot T_B(m_p)}{T_H(m_{ph})}\right)^2 \quad \textbf{(18)}$$

$$N=\left(\frac{T_H(m_p)}{8\cdot\pi\cdot T_B(m_u)}\right)^2=\left(\frac{T_H(m_u)}{8\cdot\pi\cdot T_B(m_p)}\right)^2 \quad \textbf{(19)}$$

$$N=\left(\frac{T_H(m_u)}{T_H(m_{ph})}\right)=\left(\frac{T_B(m_{ph})}{T_B(m_u)}\right) \quad \textbf{(20)}$$

$$N=\left(\frac{\beta^{1/2}\cdot T_H(m_u)}{8\cdot\pi\cdot\alpha\cdot T_B(m_e)}\right)^3=\left(\frac{\beta^{1/2}\cdot T_H(m_e)}{8\cdot\pi\cdot\alpha\cdot T_B(m_u)}\right)^3=\left(\frac{8\cdot\pi\cdot\alpha^2\cdot T_B(m_e)}{\beta\cdot T_H(m_e)}\right)^3 \quad \textbf{(21)}$$

$$N=\frac{\alpha^2\cdot T_B^2(m_e)}{\beta\cdot T_B^2(m_p)}=\frac{\alpha^2\cdot T_H^2(m_p)}{\beta\cdot T_H^2(m_e)} \quad \textbf{(22)}$$

$$N=\left(\frac{8\cdot\pi\cdot\alpha\cdot T_B(m_e)}{\beta^{1/2}\cdot T_H(m_{ph})}\right)^{3/2}=\left(\frac{8\cdot\pi\cdot\alpha\cdot T_B(m_{ph})}{\beta^{1/2}\cdot T_H(m_e)}\right)^{3/2} \quad \textbf{(23)}$$

Here is an interesting equality that we may find when we calculate the $T_B$ and $T_H$ temperatures as a function of the Planck mass $m_p$:

$$8\cdot\pi\cdot T_B(m_p)=T_H(m_p) \quad \textbf{(24)}$$

Another interesting equation is the following one:

$$T_B(m_i)\cdot T_H(m_i)=T_B(m_n)\cdot T_H(m_n) \quad \textbf{(25)}$$

Here, $m_i$ and $m_n$ are some different masses. This equation is easy to demonstrate since the product of equations **(10)** by **(13)** forces the mass $m$ to null out. The product of $T_B(m)$ by $T_H(m)$ becomes independent of the mass.

## 4. CONCLUSION

Some links exist between the Beckenstein-Hawking temperature and the Hagedorn one. The ratio of these temperatures also finds its echo in the ratio between the apparent radius of curvature of the universe $r_u$ and the classical radius of the electron $r_e$.

As the reader will have noticed, it is possible to establish a multitude of links. However, we would like to mention that all these links would not be possible without the $\beta$ constant which comes from our researches on the acceleration of light over time. These links still reinforce the hypothesis that light accelerates over time according to our model of the universe [4].

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