Large Numbers Hypothesis of Dirac Leading to the Hubble Constant and to the Temperature of the Cosmic Microwave Background

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In his large numbers hypothesis from 1974, Dirac noticed that it seems possible to establish to a parallel between the electron and the universe [11].

By setting a similar parallel between the electron rotating around a proton in the hydrogen atom and our rotating universe [1,2]*, it is possible to get an equation that gives the value of the Hubble constant H_0 as a function of the universal gravitational constant G, of the mass of the electron m_e,, of the classical radius of the electron r_e , of the fine structure constant α and of the speed of light in the vacuum c. From these constants, G is the less precise one and sets the value of the Hubble constant to $H_0 \approx 72.10 \pm 0.01$ km/(s·MParsec). This value is in accordance with the value of $H_0 \approx 72.1 \pm 0.9$ km/(s·MParsec) obtained by Xiaofeng Wang and his team* [16].

In a previous work, we found an equation that was linking H_0 to the temperature T of the cosmic microwave background (CMB) [5]. Based on the fact that D. J. Fixsen [3] *found that $T \approx 2.72548 \pm 0.00057$ ºK, we obtained $H_0 \approx 71.50 \pm 0.01$ km/(s·MParsec). Making equal the two equations which give H_0, it becomes possible to isolate the variable T and to evaluate it precisely as a function of G (up to now, this constant is the most imprecise one). Therefore, we obtain a value of $T \approx 2.7369 \pm 0.0002$ ºK. This temperature is compatible with some measurements that have been taken between 1982 and 1990 by the Cobra rocket* [4] *that evaluated the temperature of the CMB to 2.736 ±0,017 ºK.* [1]

KEY WORDS: Dirac, Hubble constant, temperature of the universe, CMB, electron

1. INTRODUCTION

Making calculations about the universe, Dirac found that the ratio of certain numbers always seemed to lead to specific numbers. Without finding all the secrets linking them, these numbers seem interrelated. Dirac published a paper on his large numbers hypothesis.

Several studies in astrophysics are based on the knowledge of the Hubble constant. This allows, among others, to calculate the apparent age of the universe and to know the recession speed of galaxies relative to us according to their

[1] Recent works that we make allowed us to evaluate the Hubble constant to $H_0 \approx 72.09548632 \pm 0{,}000000046$ km/(s·MParsec) and the universal gravitational constant to $G \approx 6.67323036 \pm 0{,}00000030 \times 10^{-11}$ m^3/(kg·s^2). This situation allowed to revaluate the average temperature of the CMB to $T \approx 2.7367951 \pm 0.0000026$ °K [18].

distance [17]. That is why we are interested in knowing this value as accurately as possible.

The purpose of this paper is to find a new equation, independent of the recently discovered ones [5], which would allow us to calculate the Hubble constant independently of the temperature T of the cosmic microwave background (CMB). We will see, like Dirac, that the ratio of some numbers seems to act like scale factors. Knowing this scale factor, it becomes possible for us to generate a new equation for the Hubble constant. Using the equation found in our first paper on the Hubble constant [5] and the new equation of this document, we make a system with two equations and two unknowns (including the Hubble constant H_0 and the temperature of the cosmic microwave background T). It becomes possible for us to determine accurately these parameters based on the universal gravitational constant G and other known physical constants. This procedure is used to determine H_0 and T as the same uncertainty than the one for G.

This method will lead us to complete some missing elements in the large number hypothesis of Dirac. We will transform some equations that led him to see similarities between certain relationships or equations of the same order of magnitude in such a way that we will achieve equality between them.

2. DEVELOPEMENT

2.1. Theoretical Hubble Constant from Previous Studies

In previous studies, we obtain an equation which allows the calculation of the Hubble constant H_0 depending on the average cosmic microwave background (CMB) temperature [12]. Based on the average CMB temperature (2.72548 ºK) presented by Fixsen [6], we get the following Hubble constant:

$$H_0 = \frac{\pi^3 \cdot T^2 \cdot k_B^2 \cdot \sqrt{\dfrac{8 \cdot G}{15 \cdot c^5 \cdot h^3}}}{\beta^2 \cdot \alpha} \approx 71.50 \pm 0.03\,\text{km/(s·MParsec)} \quad \textbf{(1)}$$

These following values come from the CODATA 2010 [6]:

- Planck constant $h \approx 6.62606957\times10^{-34}$ J·s
- Actual speed of light in vacuum $c \approx 299792458$ m/s
- Universal gravitational constant $G \approx 6.67384\times10^{-11}$ $m^3/(kg\cdot s^2)$
- Fine structure constant $\alpha \approx 7.2973525698\times10^{-3}$
- Boltzmann constant $k_B \approx 1.3806488\times10^{-23}$ J/ºK

Regarding the β constant, it represents the relationship between the rate of expansion of the material universe and the speed of light [7]. The value of β is a pure number.

$$\beta = 3 - \sqrt{5} \approx 0.764 \tag{2}$$

The uncertainty shown in equation (1) comes from the uncertainty in the average CMB temperature, according to Fixsen [3] which is ± 0.00057 ºK. As we shall see later, this uncertainty is probably underestimated. However, the value of H_0 obtained in **(1)** is consistent with that measured by the David Rapetti team [13] which obtained $H_0 \approx \pm 1.3$ 71.5 km / (s · MParsec).

2.2. Coincidence Noted by Dirac

In order to obtain eventually a second equation giving the value of the Hubble constant, let's analyze one of the coincidences discovered by Dirac in 1973 [11].

Using the value of H_0 shown in **(1)**, the mass of the apparent observable mass of the universe is equal to [8,9]:

$$m_u = \frac{c^3}{G \cdot H_0} \approx 1.73 \times 10^{53}\,\mathrm{kg} \tag{3}$$

Assuming that the luminous universe [7] is currently expanding at the speed of light in vacuum c [10], the apparent radius of curvature of the luminous universe would be:

$$R_u = \frac{c}{H_0} \approx 1.3 \times 10^{26}\,\mathrm{m} \tag{4}$$

The largest possible unit of distance in the universe becomes its circumference. Therefore, the smallest mass that can exist is the one that can be associated with a photon of wavelength $2 \cdot \pi \cdot R_u$:

$$m_{ph} = \frac{h}{2 \cdot \pi \cdot R_u} = \frac{h \cdot H_0}{2 \cdot \pi} \approx 2.72 \times 10^{-69}\,\mathrm{kg} \tag{5}$$

Let's consider the number N as being the maximum number of photons of wavelength $2 \cdot \pi \cdot R_u$ in the universe. If the universe has a mass m_u, and the photon, the mass m_{ph}, the number N is equal to:

$$N = \frac{m_u}{m_{ph}} = \frac{2 \cdot \pi \cdot c^5}{G \cdot h \cdot H_0^2} \approx 6.41 \times 10^{121} \quad \textbf{(6)}$$

In 1973, Dirac published a hypothesis on large numbers [11]. Assuming that the mass m_e of an electron comes from its electrostatic energy E_e and its gravitational energy E_g, Dirac made the analysis of the relationship between these two forms of energy to know the relative contribution of each. He found that the contribution of the electrostatic energy was about 4×10^{42} times greater than the gravitational energy.

$$\frac{E_e}{E_g} = \frac{q_e^2}{4 \cdot \pi \cdot \varepsilon_0 \cdot G \cdot m_e^2} \approx 4.2 \times 10^{42} \quad \textbf{(7)}$$

According to the CODATA 2010 [6], the value of the permittivity of the vacuum is $\varepsilon_0 \approx 8.854187817$ F/m.

Dirac then found that this number is in about the same order of magnitude as $N^{1/3}$:

$$N^{1/3} = \left(\frac{2 \cdot \pi \cdot c^5}{G \cdot h \cdot H_0^2} \right)^{1/3} \approx 4.0 \times 10^{40} \quad \textbf{(8)}$$

He made the assumption that there is a proportional relationship between equations **(7)** and **(8)**.

His finding was only a hypothesis based on similar orders of magnitude and certainly lacked accuracy (note that there is a factor of about 100 between equations **(7)** and **(8)**). It therefore could not be used as it is, but it could serve to suspect a potential proportionality link.

2.3. Mass of the Electron Rotating Around a Proton in an Hydrogen Atom

To make certain calculations, we will eventually need to know the mass m'_e of the electron rotating around a proton in the hydrogen atom.

The mass m'_e depends on the rotation speed around the proton. As it is in rotation at a speed that is close to the speed of light, its mass in rotation is obtained by dividing its rest mass m_e by the Lorentz factor [14]. Here, the rotation speed is represented by ω:

$$m'_e = \frac{m_e}{\sqrt{1-\frac{\omega^2}{c^2}}} \tag{9}$$

We need to know the value of the speed ω.

Let's begin by making the following assumptions:

- The electron rotates on itself at a speed ω.
- The electron rotates around the proton at a speed ω.
- The fine structure constant α is in fact the following Lorentz factor:

$$\alpha = \sqrt{1-\frac{\omega^2}{c^2}} \tag{10}$$

The rotation speed ω would therefore be the following:

$$\omega = c \cdot \sqrt{1-\alpha^2} \approx 0.999973 \cdot c \tag{11}$$

Using the equations **(9)** and **(10)**, we obtain the value of m'_e:

$$m_e{}' = \frac{m_e}{\sqrt{1-\frac{\omega^2}{c^2}}} = \frac{m_e}{\alpha} \tag{12}$$

Knowing that the Compton radius r_c of an electron is in fact the apparent radius of an electron when it spins to a speed close to the one of light (it is a relativistic phenomenon explained by Einstein with spinning disks [15]), we obtain the following relationship:

$$r_c = \frac{r_e}{\sqrt{1-\frac{\omega^2}{c^2}}} = \frac{r_e}{\alpha} \approx 3.86 \times 10^{-13}\text{m} \tag{13}$$

If, in addition to spinning on itself, the electron orbits around the proton (the hydrogen nucleus) at a speed ω, another α factor will add up to give the average rotation distance between electron-proton in the hydrogen atom. We will name this distance r_B (Bohr radius). For calculation purposes, we will use the classical value of the radius r_e and the fine structure constant α mentioned in the CODATA 2010 [6] :

$$r_B = \frac{r_c}{\alpha} = \frac{r_e}{\alpha^2} = \frac{r_e}{1-\frac{\omega^2}{c^2}} \approx 0.52917721092 \times 10^{-10}\,\text{m} \tag{14}$$

Where $r_e \approx 2.8179403267 \pm 0.0000000027 \times 10^{-15}\,\text{m}$ (CODATA)

$\alpha \approx 7.2973525698 \pm 0.0000000024 \times 10^{-3}$ (CODATA)

The value of the equation **(14)** is exactly the average distance measured in the hydrogen atom (level 1) that is mentioned in the CODATA 2010 under the name of « Bohr radius » [6].

$$r_B \approx 0.52917721092 \pm 0.00000000017 \times 10^{-10}\,\text{m} \tag{15}$$

This confirms that our hypotheses are probably exact and therefore equations **(10)**, **(12)** and **(14)** are correct.

2.4. Hypothesis on Mass/Radius Ratios in the Universe and in the Electron

In a manner similar to that of Dirac [11], we noticed that the $N^{1/3}$ factor seems to be a scale factor between two well-defined ratios:

1) The ratio between the apparent mass of the universe in rotation m_u and the apparent curvature radius r_u at our location in the universe.
2) The ratio between the mass of the electron m'_e rotating around the proton in the hydrogen atom and the classical radius of the electron r_e.

So, we have:

$$\frac{\left(\frac{m_u}{r_u}\right)}{\left(\frac{m'_e}{r_e}\right)} = N^{1/3} \approx 4.0 \times 10^{40} \tag{16}$$

Here, the value of the apparent curvature radius r_u is evaluated at our location in the universe. Since matter is moving more slowly than light, the progression of the material universe expansion is slower than the luminous universe. In fact, according to our calculation [7], it is moving at a speed equal to $\beta \cdot c$ (see equation **(2)** to know the value of β). Therefore:

$$r_u = \frac{\beta \cdot c}{H_0} = \beta \cdot R_u \approx 1.0 \times 10^{26}\,\mathrm{m} \tag{17}$$

So, the equation **(16)** becomes:

$$\frac{\left(\dfrac{m_u}{\beta \cdot R_u}\right)}{\left(\dfrac{m'_e}{r_e}\right)} = N^{1/3} \approx 4.0 \times 10^{40} \tag{18}$$

The mass m'_e of equation **(18)** represents the mass of the electron rotating around the nucleus. As shown at equation **(12)**, it includes a Lorentz factor (with a speed ω) because of its rotating speed around it. Therefore, using equation **(12)**, we obtain:

$$\frac{\left(\dfrac{m_u}{\beta \cdot R_u}\right)}{\left(\dfrac{m_e}{r_e \cdot \alpha}\right)} = \frac{m_u \cdot r_e \cdot \alpha}{m_e \cdot R_u \cdot \beta} = N^{1/3} \approx 4.0 \times 10^{40} \tag{19}$$

Although for now we cannot prove the veracity of equations **(16)** and **(19)**, they seem logical and natural. Indeed, these ratios refer to similar parameters within two different orders of magnitude: one for the rotating universe and the other for the electron in the hydrogen atom.

Let's replace m_u by the equation **(3)** and R_u by the equation **(4)** in the equation **(19)**:

$$\frac{c^2 \cdot r_e \cdot \alpha}{G \cdot m_e \cdot \beta} = N^{1/3} \approx 4.0 \times 10^{40} \tag{20}$$

Using the equation **(8)** in the equation **(20)**, se obtain:

$$\frac{c^2 \cdot r_e \cdot \alpha}{G \cdot m_e \cdot \beta} = \left(\frac{2 \cdot \pi \cdot c^5}{G \cdot h \cdot H_0^2}\right)^{1/3} \approx 4.0 \times 10^{40} \tag{21}$$

Rearranging the equation **(21)**, we obtain:

$$\frac{m_e \cdot \beta}{r_e \cdot \alpha} \cdot \left(\frac{2 \cdot \pi \cdot G^2}{h \cdot H_0^2 \cdot c} \right)^{1/3} \approx 1.006 \tag{22}$$

If we make a numerical evaluation of equation **(22)**, the result is close enough to 1 so that we can make the hypothesis that it should equal 1 if we had a better value of G. Effectively, it is the less precise constant of all of them. Therefore, we will make that conjecture.

2.5. Calculation of the Theoretical Hubble Constant H_0

In equation **(22)**, the parameter that has the largest uncertainty is the Hubble constant H_0 because the others are known more precisely. Therefore, let's isolate this parameter taking into account the accuracy of G (which has the largest uncertainty):

$$H_0 = G \cdot \sqrt{\frac{2 \cdot \pi \cdot m_e^3 \cdot \beta^3}{h \cdot c \cdot r_e^3 \cdot \alpha^3}} \tag{23}$$

Let's associate the energy contained in the mass at rest of the electron to a wave. We must use the Compton radius r_c to calculate the wavelength. Using equation **(13)**, we obtain:

$$m_e \cdot c^2 = \frac{h \cdot c}{2 \cdot \pi \cdot r_c} = \frac{h \cdot c}{2 \cdot \pi \cdot \left(\frac{r_e}{\alpha} \right)} \tag{24}$$

By doing a few algebraic replacements in **(23)** with the help of the equation **(24)**, we obtain:

$$H_0 = \frac{G \cdot m_e \cdot \beta^{3/2}}{r_e^2 \cdot \alpha \cdot c} \approx 72.10 \pm 0.009 \frac{\text{km}}{\text{s} \cdot \text{MParsec}} \tag{25}$$

This value is in agreement with the value obtained by the Xiaofeng Wang research team which was $H_0 \approx 72.1 \pm 0.9$ km/(s·MParsec). It is also consistent with the value obtained by the David Rapetti team [13] which was $H_0 \approx 71.5 \pm 1.3$ km/(s·MParsec).

Let's note that the precision of the value obtained in **(25)** mainly depends on G.

2.6. Calculation of the Average Temperature *T* of the Cosmic Microwave Background

In previous work, we managed to make a theoretical link between the Hubble constant H_0 and the temperature of the cosmic microwave background (CMB) *T* [12]. We then obtained the equation (1). Note that the value of the Hubble constant obtained by this equation is different from that obtained with the equation **(25)**. Although Fixsen and his team have obtained 2.72548 ± 0.00057 ºK, we believe that the experimental evaluation of the temperature of the CMB that was used to calculate the value of the Hubble constant in **(1)** may not be as precise as what the authors of the study suggest. The relative error of the obtained temperature is about twice as large as the universal gravitational constant *G*. The fact remains that both are in about the same order of magnitude.

If we give more credibility to the universal gravitational constant *G* value than to the temperature obtained by Fixsen, we could get the average temperature of the cosmologic microwave background *T* by evaluating it on a theoretical basis, inter alia, on the constant *G*. To do this, we will match the equations **(1)** and **(25)** and isolate *T*. We will also use the equality shown in equation **(24)**. We then obtain:

$$T = \frac{1}{k_B} \cdot \left(\frac{15 \cdot G \cdot m_e^5 \cdot c^6 \cdot \beta^7}{\pi^3 \cdot r_e \cdot \alpha^3} \right)^{1/4} \approx 2.73686 \pm 0.00008 °\text{K} \tag{26}$$

This temperature value agrees well with the measurements obtained by the COBRA space probe [4] which obtained 2.736 ± 0.0017 ºK. The value of *T* obtained in **(25)** depends mainly on the accuracy of the universal gravitational constant *G*. So it seems that our assumptions for the calculation are correct.

3. COMPARISON OF RESULTS WITH THE HYPOTHESIS OF DIRAC

If we make the hypothesis that the mass at rest of the electron is entirely due to its electrostatic energy E_s, we obtain the following equation:

$$E_0 = m_e \cdot c^2 = \frac{q_e^2}{4 \cdot \pi \cdot \varepsilon_0 \cdot r_e} \tag{27}$$

We reorganize the equation **(16)** by using the equation **(27)** to obtain:

$$\frac{q_e^2 \cdot \alpha}{4 \cdot \pi \cdot \varepsilon_0 \cdot G \cdot m_e^2 \cdot \beta} = N^{1/3} \approx 4.0 \times 10^{40} \tag{28}$$

If we take this result and compare it to equation **(7)**, we see that there is a factor α at the numerator and a factor β at the denominator. These are exactly the required factors to make the Dirac equation equal to $N^{1/3}$.

Then, we could wonder where the α and β constants come from.

Let's suppose that we analyze the kinetic energy E_k of an electron traveling collinearly with another electron at a speed ω. Let's suppose that both electrons are distanced, with respect to each other, with a distance r.

According to Einstein's relativity [15], the total kinetic energy E_k of the electron having a mass m_e and traveling at a speed ω is given by:

$$E_k = \frac{E_0}{\sqrt{1-\frac{\omega^2}{c^2}}} = \frac{m_e \cdot c^2}{\sqrt{1-\frac{\omega^2}{c^2}}} \tag{29}$$

Using the equation **(12)**, the equation **(29)** becomes:

$$E_k = \frac{m_e \cdot c^2}{\alpha} \tag{30}$$

If we use the equation **(27)** in the particular case where $r = r_e$, the equation (30) becomes:

$$E_k(r_e) = \frac{m_e \cdot c^2}{\alpha} = \frac{q_e^2}{4 \cdot \pi \cdot \varepsilon_0 \cdot r_e \cdot \alpha} \tag{31}$$

Generalizing a bit more the equation **(31)** to put it as a function of r, we obtain:

$$E_k(r) = \frac{m_e \cdot c^2}{\alpha} = \frac{q_e^2}{4 \cdot \pi \cdot \varepsilon_0 \cdot r \cdot \alpha} \tag{32}$$

Similarly as for the total kinetic energy of an electron, let's analyze the gravitational energy $Eg(r)$ for a moving electron of mass m_e separated by a distance r with respect to another electron. The two electrons travel collinearly, with a speed ω.

If electrons were static, the gravitational energy E_g would have been:

$$E_g(r) = \frac{G \cdot m_e^2}{r} \text{ (static case)} \tag{33}$$

But, as they are moving at a speed ω, each mass is affected by the relativistic Lorentz factor for a speed ω. Therefore, we obtain:

$$E_g(r) = \frac{G \cdot m_e^2}{r \cdot \left(1 - \frac{\omega^2}{c^2}\right)} \text{ (dynamic case)} \tag{34}$$

Using the equation **(12)**, we obtain:

$$E_g(r) = \frac{G \cdot m_e^2}{r \cdot \left(1 - \frac{\omega^2}{c^2}\right)} = \frac{G \cdot m_e^2}{r \cdot \alpha^2} \text{ (dynamic case)} \tag{35}$$

Now, if we set the ratio between the kinetic energy of the electron and the gravitational energy, we obtain:

$$\left.\frac{E_k(r)}{E_g(r)}\right|_{r = r_u} = \frac{q_e^2 \cdot \alpha}{4 \cdot \pi \cdot \varepsilon_0 \cdot G \cdot m_e^2} = \beta \cdot N^{1/3} \approx 4.165 \times 10^{42} \tag{36}$$

If we analyze this equation, we see that this ratio applies only here, at r_u, in the material universe. Effectively, we made the calculation with the constants that are known here, on Earth. This is why there is a β constant in front of the $N^{1/3}$ factor. The material universe must necessarily expand more slowly than the speed of light. Consequently, our position r_u is at β times the apparent radius of curvature R_u of the luminous universe.

If we isolate $N^{1/3}$ to know the ratio that subsists at the edges of the luminous universe (at R_u), we have:

$$\left.\frac{E_k(r)}{E_g(r)}\right|_{R_u} = \frac{q_e^2 \cdot \alpha}{4 \cdot \pi \cdot \varepsilon_0 \cdot G \cdot m_e^2 \cdot \beta} = N^{1/3} \approx 3.979 \times 10^{40} \tag{37}$$

This last equation explains the similarity observed by Dirac between equations **(7)** and **(8)**. We note that the only differences are the α and β factors that appear in the numerator and denominator. Therefore, for the intuition of Dirac to be realized, we can, among other things, evaluate the ratio between the kinetic energy and the gravitational energy of an electron moving collinearly with another electron at a speed ω. We would have the same result by evaluating the ratio between the relativistic electric force versus the relativistic gravitational force between two electrons traveling collinearly at a speed ω.

4. CONCLUSION

We note that the large numbers hypothesis of Dirac [11] seems to have a real foundation. Some large numbers are used as scale factor. One of these large numbers, $N^{1/3}$, seems useful to determine the scale factor linking the mass/radius of the universe to the mass/radius of a rotating electron in the hydrogen atom. It also seems related to the electrostatic/gravitational acceleration ratio between two electrons.

Indirectly, the large numbers hypothesis of Dirac led us to find an equation that now allows us to calculate the Hubble constant H_0 accurately based primarily on the universal gravitational constant G. At the same time, making the connection with an equation found in previous work has enabled us to find the temperature T of the cosmologic microwave background (CMB) based primarily on the universal gravitational constant G.

In the future, should we find an independent way to calculate H_0 or T, it will be possible to calculate precisely the universal gravitational constant according to highly more accurate parameters than G is for now.

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