The Speed of Light, Masses, Dimensions and Time Are Influenced by Gravitational Fields

Claude Mercier eng., February 2nd 2015 claude.mercier@cima.ca
Rev. October 17th 2015

The principle of conservation of mass that comes from classical mechanics is commonly used in physics and chemistry. However, this principle is only an approximation of the reality and is valid only in a given specific framework. Indeed, just as in Newton's equations, this principle ignores the relativistic effects. It is valid provided the speeds of the objects involved are relatively low and the surrounding gravitational fields are relatively small.

Thanks to the equations of the special relativity, Einstein showed, in 1911, that the speed of light could be influenced by a gravitational field [8]. However, even though the idea he enunciated at the time was good, the equation that he found through special relativity had to be rectified subsequently by a factor of 2 when he worked on the general relativity. Thanks to the general relativity, Schwarzschild showed that the speed of light decreases in the presence of an intense gravitational field [5,6]. This is also the basis for the existence of gravitational lenses which have been widely tested since [10,11].

Knowing that Newton's equations on the potential energy are valid at low speed and using Schwarzschild's equation [5,6] which gives the speed of light as a function of the surrounding gravitational field, it is possible to show that a mass subjected to a gravitational field sees its relative value (relative to the mass outside any gravitational field) decrease due to the change in the index of refraction of the vacuum. We believe that this same phenomenon (purely relativistic) also applies to the time and space dimensions. We will show that the masses and time are inversely proportional to the index of refraction of vacuum n just like the speed of light. The dimensions of the space, in turn, are directly proportional to n.

KEY WORDS: Speed of light, mass, dimensions, time, gravitational field, index of refraction of the vacuum, potential energy

1. INTRODUCTION

In this document, we will start from the Schwarzschild equation [5,6] which gives the speed of light depending on the surrounding gravitational field and we will modify this equation to make it look like the potential energy variation equation of Newton. Doing so, it will be possible to show that the mass of objects varies with the inverse of the index of refraction of the vacuum. As for the special relativity theory, we will extrapolate this principle to the space and time dimensions.

2. DEVELOPMENT

In 1911, Einstein showed that the speed of light in vacuum is influenced by the presence of a gravitational field [8]. The gravitational potential Φ is modulated as a function of the mass m_1 and of the distance r_1 with respect to the latter. The closer we are to the mass m_1, the more intense is the gravitational field. As stated in equation 3 of his article titled ″On the Influence of Gravitation on the Propagation of Light″, Einstein came to this result:

$$c = c' \cdot \left(1 + \frac{\Phi}{c^2}\right) \quad \text{(Equation based on the special relativity)} \tag{1}$$

The gravitational potential Φ is given by the following equation:

$$\Phi = \frac{G \cdot m_1}{r_1} \tag{2}$$

According to studies published previously [9], the precise value of the gravitational constant is related to the value of the classical electron radius r_e, the electron mass m_e, the fine structure constant α, the speed of light in vacuum c and β.

$$G = \frac{c^2 \cdot r_e \cdot \alpha^{20}}{m_e \cdot \beta} \approx 6.673230436(30) \times 10^{-11}\ \mathrm{m}^3/\left(\mathrm{kg} \cdot \mathrm{s}^2\right) \tag{3}$$

According to the CODATA 2010 [1]:

- Universal gravitational constant $G \approx 6.67384(80) \times 10^{-11}\ \mathrm{m}^3/(\mathrm{kg} \cdot \mathrm{s}^2)$
- Speed of light in vacuum $c \approx 299792458$ m/s
- Radius of the classical electron $r_e \approx 2.8179403267(27) \times 10^{-15}$ m
- Mass of the electron $m_e \approx 9.10938291(40) \times 10^{-31}$ kg
- Fine structure Constant $\alpha \approx 7.2973525698(24) \times 10^{-3}$

The value of β is an irrational number. It gives the ratio between the expansion speed of the material universe and the speed of light in the vacuum c [4]:

$$\beta = 3 - \sqrt{5} \approx 0.76 \tag{4}$$

More accurately, according to the Schwarzschild calculations which use the general relativity of Einstein, the speed of light c' varies according to the index of refraction of the vacuum n_1 as shown in the following equation:

$$c' = \frac{c}{n_1} \quad \text{où} \quad n_1 = \sqrt{\frac{1 + \frac{2\Phi}{c^2}}{1 - \frac{2\Phi}{c^2}}} \tag{5}$$

This result has been amply verified through gravitational lensing [10,11] and is considered accurate by the scientific community.

For a relatively low gravitational potential Φ, the refractive index n_1 can be approximated as follows:

$$n_1 \approx 1 + \frac{2\Phi}{c^2} \tag{6}$$

We notice that Einstein, basing himself only on the special relativity, came to a result that does not correspond to the physical reality (contrarily to the general relativity) by a factor 2. Effectively, using the equations **(5)** and **(6)** as a basis, he should have come, with good approximation, to the following equation:

$$c \approx c' \cdot \left(1 + \frac{2\Phi}{c^2}\right) \tag{7}$$

Now, let's suppose a relatively light mass m_2 in the presence of an imposing mass m_1 which generates a gravitational potential Φ. Let's try to establish a link between mass m_2 and its potential energy with respect to mass m_1.

Let's begin by modifying the equation **(5)** in this way:

$$n_1^2 = \frac{1 + \dfrac{2G \cdot m_1}{c^2 \cdot r_1}}{1 - \dfrac{2G \cdot m_1}{c^2 \cdot r_1}} \tag{8}$$

We can rewrite this equation as follows:

$$n_1^2 \cdot \left(1 - \frac{2G \cdot m_1}{c^2 \cdot r_1}\right) = 1 + \frac{2G \cdot m_1}{c^2 \cdot r_1} \tag{9}$$

With some algebraic manipulations, we have:

$$\frac{n_1^2 - 1}{n_1^2 + 1} = \frac{2G \cdot m_1}{c^2 \cdot r_1} \tag{10}$$

As the index of refraction is always greater than or equal to 1, we can say that:

$$n_1 = 1 + \varepsilon \tag{11}$$

Therefore, we can rewrite the left side of equation **(10)** as follows:

$$\frac{n_1^2-1}{n_1^2+1}=\frac{(1+\varepsilon)^2-1}{(1+\varepsilon)^2+1}=\frac{1+2\varepsilon+\varepsilon^2-1}{1+2\varepsilon+\varepsilon^2+1}=\frac{2\varepsilon+\varepsilon^2}{2+2\varepsilon+\varepsilon^2} \tag{12}$$

For the numerator, with ε of a small value, the term ε^2 is negligible. In good approximation, the equation **(12)** gives:

$$\frac{n_1^2-1}{n_1^2+1}\approx\frac{2\varepsilon}{2+2\varepsilon}=\frac{\varepsilon}{1+\varepsilon} \tag{13}$$

Using the equation **(11)**, the equation **(13)** becomes:

$$\frac{n_1^2-1}{n_1^2+1}\approx\frac{n_1-1}{n_1}=\frac{1}{1}-\frac{1}{n_1} \tag{14}$$

Let's rewrite the equation **(10)** using the equation **(14)**.

$$\frac{1}{1}-\frac{1}{n_1}\approx\frac{2G\cdot m_1}{c^2\cdot r_1} \tag{15}$$

By doing some algebraic manipulations, we obtain:

$$\left(\frac{1}{1}-\frac{1}{n_1}\right)\cdot\frac{c^2}{2}\approx\frac{G\cdot m_1}{r_1} \tag{16}$$

Now, let's try to make the right side of equation **(16)** look like the following Newton equation which gives the potential energy variation ΔE of an object of mass m_2 starting from a distance r_1 with respect to the center of mass m_1 and moved to an infinite distance r_2 with respect to mass m_1.

$$\Delta E\Big|_{r_1}^{r_2}=\lim_{r_2\to\infty}\left(\frac{-G\cdot m_1\cdot m_2}{r_2}-\frac{-G\cdot m_1\cdot m_2}{r_1}\right) \tag{17}$$

Let's make the value of the mass m_2 in equation **(16)** appear on each side of the equation.

$$\left(\frac{1}{1}-\frac{1}{n_1}\right)\cdot\frac{m_2\cdot c^2}{2}\approx-\frac{-G\cdot m_1\cdot m_2}{r_1} \tag{18}$$

We know that in the equation **(17)**, the following value is null:

$$\lim_{r_2\to\infty}\frac{-G\cdot m_1\cdot m_2}{r_2}=0 \tag{19}$$

Therefore, we can add it without changing the right side of equation **(18)**.

$$\frac{1}{2}\cdot\left(\frac{1}{1}-\frac{1}{n_1}\right)\cdot m_2\cdot c^2 \approx \lim_{r_2\to\infty}\left(\frac{-G\cdot m_1\cdot m_2}{r_2}-\frac{-G\cdot m_1\cdot m_2}{r_1}\right) \quad \textbf{(20)}$$

When we analyse the right side of the equation **(20)**, we see in the equation of the potential energy variation that an object of mass m_2 would gain by moving to a distance r_2 which tends toward infinity from an initial distance r_1 with respect to a mass m_1.

According to the special relativity of Einstein, the total energy of mass m_2 at rest is given by the following transformation equation:

$$E = m_2\cdot c^2 \quad \textbf{(21)}$$

The index of refraction of the vacuum, for a position beyond any gravitational field (at a distance $r_2\to\infty$) is "1" and the index of refraction of the vacuum at the position r_1, relative to the mass m_1, is n_1. To get a change in the potential energy, the energy contained in the mass m_2 is forced to change according to its distance from the center of mass m_1. In other words, the energy E of the mass m_2 varies depending on half the difference between the inverse of the index of refraction of the vacuum at $r_2\to\infty$, i.e. 1, and the inverse of the index of refraction r_1, that is to say n_1.

In the equation **(20)**, we analysed the case of a mass m_2 that starts from a distance r_1 with respect to the mass m_1 and moves at a distance $r_2\to\infty$. However, for a more general case, it is possible to show mathematically that we could express the equation **(20)** as follows:

$$\Delta E\Big|_{r_1}^{r_2} = \frac{-G\cdot m_1\cdot m_2}{r_2}-\frac{-G\cdot m_1\cdot m_2}{r_1} \approx \frac{1}{2}\cdot\left(\frac{1}{n_2}-\frac{1}{n_1}\right)\cdot m_2\cdot c^2 \quad \textbf{(22)}$$

This equation tells us that a potential energy variation ΔE, between the points r_1 and r_2, of the mass m_2 (which moves with respect to the center of mass m_1), is approximately equal to half the variation of the energy contained in the matter of the mass m_2 influenced by the index of refraction of the vacuum of its environment (n_1 for the point located in r_1 and n_2 to the point r_2).

For an event of short duration, the speed limit of light in the vacuum is considered constant. For sure, this is not the case for events of long duration since we claim that the speed of light increases over time [4] because of the expansion of the universe [7]. Because the universe is expanding, the index of refraction of the vacuum decreases over time [4]. The speed of light will eventually tend toward a value that we baptized k for an apparent radius of curvature of the universe R_u that will tend toward infinity [4].

$$k = c \cdot \sqrt{2+\sqrt{5}} \approx 2 \cdot c \tag{23}$$

The apparent radius of the universe varies according to the Hubble constant H_0. According to previous works [12], the apparent radius of the universe R_u can be expressed more accurately by using the following equation:

$$R_u = \frac{c}{H_0} = \frac{r_e}{\beta^{1/2} \cdot \alpha^{19}} \approx 1.2831078806(81) \times 10^{26}\,\mathrm{m} \tag{24}$$

In previous work, we have already shown that the Hubble constant H_0 could be expressed by an equation whose accuracy depended on the speed of light c, the fine structure constant α, the classical radius of the electron r_e and the β constant (which is described in equation **(4)**).

$$H_0 = \frac{c \cdot \alpha^{19} \cdot \beta^{1/2}}{r_e} \approx 72.09548632(46)\ \mathrm{km}/\left(s \cdot MPar\,\mathrm{sec}\right) \tag{25}$$

The value of the Hubble constant H_0 obtained in **(25)** is compatible with the one Xiaofeng Wang and his team [13] obtained as in the following measurement: H_0 = 72.1 ±0.9 km/(s·MParsec).

We could be tempted to believe, according to the equation **(21)**, that if the speed of light c increases over time, the energy of matter increases over time. But this is not true. The law of energy conservation continues to apply and the energy stays the same. Furthermore, to maintain the energy of matter over time, the mass of objects diminishes consequently over time [14].

3. VARIATION OF THE MASS OF OBJECTS AS A FUNCTION OF THE INDEX OF REFRACTION OF THE VACUUM

Let's try to see what should be the impact of the variation of the index of refraction in the vacuum on the mass of objects. Let's begin by setting up an analogy between the nuclear fusion reaction and the gravitational attraction. In fact, for any force of attraction, the fact of moving the objects closer together forces them to lose potential energy and lose mass. In the process of moving the masses all together, there is always emission of photons, even in gravitation.

Experimental measurements show that the mass of an atomic nucleus is always less than that of its constituents taken separately and added together. Let's take a simple random example, ^{14}C carbon. We deliberately take a carbon isotope as an example to avoid the comparison with the average atomic mass figuring in the Mendeleyev's periodic table. Indeed, this value reflects a weighted average of the percentage found naturally in nature.

Table 1: Calculation of the sum of the carbon constituents masses of ^{14}C			
Constituents	Quantity	Mass (kg)/particle	Total mass (kg)
Protons	6	$1.672621777\times10^{-25}$ kg	$1.0035730662\times10^{-26}$ kg
Neutrons	8	$1.674927351\times10^{-27}$ kg	$1.3399418808\times10^{-26}$ kg
Electrons	6	9.10938291×10^{-31} kg	$5.465629746\times10^{-30}$ kg
		Total of m_c	$2.344061510\times10^{-26}$ kg

Notwithstanding the calculation result of the sum of the masses of the ^{14}C carbon components, the total mass of this atom is $2.325293014\times10^{-26}$ kg. We see very well in this example that the mass of the atom is slightly less than the sum of the masses of the components taken separately (see Table 1). The mass gap between the mass of the ^{14}C atom (which we call here m_a) and the total mass of the components (which we call here m_c) is given by $\Delta m = m_c - m_a \approx 1.8768496215\times10^{-28}$ kg. When we force the constituents to fuse together to create a ^{14}C carbon atom, there is a mass loss Δm and a release of energy E in the form of photons. This energy E can be evaluated using the following equation of Einstein:

$$E = \Delta m \cdot c^2 \approx 1.686828317\times10^{-11}\,\text{J} \approx 105.8\,\text{MeV} \tag{26}$$

We realize that the carbon nucleus is the result of an exothermal nuclear fusion reaction. In fact, it is the same for all atoms of the Mendeleyev table.

In the process of nuclear fusion, as in gravitation, the grouping of objects happens naturally because there is attraction. However, we can imagine that when there is electrical repulsion between two particles of the same sign, grouping electric charges increases the overall mass of the objects and there is an absorption of photons.

Considering only the influence of the index of refraction on mass m_2, let's try to create an equation that highlights the simple mass variation due to the different indexes of refraction in r_1 and r_2 positions. Let's start by slightly modifying the equation **(22)** like this:

$$\frac{-G \cdot m_1 \cdot m_2}{r_2} - \frac{-G \cdot m_1 \cdot m_2}{r_1} \approx \left(\frac{1}{n_2} - \frac{1}{n_1}\right) \cdot m_2 \cdot c^2 - \frac{1}{2} \cdot \left(\frac{1}{n_2} - \frac{1}{n_1}\right) \cdot m_2 \cdot c^2 \tag{27}$$

Even if this equation seams larger than the equation **(22)**, we have changed nothing from an algebraic point of view.

We modify the appearance of the previous equation to get:

(28)

$$\underbrace{\left(\frac{1}{n_2}-\frac{1}{n_1}\right)\cdot m_2\cdot c^2}_{\text{Variation of the energy contained in matter}} \approx \underbrace{\frac{-G\cdot m_1\cdot m_2}{r_2}-\frac{-G\cdot m_1\cdot m_2}{r_1}}_{\text{Variation of potential energy}}+\underbrace{\frac{1}{2}\cdot\left(\frac{1}{n_2}-\frac{1}{n_1}\right)\cdot m_2}_{\text{Additional energy supply under photon form}}\cdot c^2$$

This equation says that the energy variation contained in the material between points 1 and 2 is due to two things: a potential energy change and an energy intake in photonic form. If this input is negative, the process emits photons. Besides, when m_2 is approaching m_1, this is what happens.

Let's divide each side of the equation **(28)** by c^2 to obtain only mass variations on each side of the equation:

$$\left(\frac{1}{n_1}-\frac{1}{n_2}\right)\cdot m_2 \approx \frac{-G\cdot m_1\cdot m_2}{r_2\cdot c^2}-\frac{-G\cdot m_1\cdot m_2}{r_1\cdot c^2}+\frac{1}{2}\cdot\left(\frac{1}{n_1}-\frac{1}{n_2}\right)\cdot m_2 \qquad \textbf{(29)}$$

This equation means that if we take a mass m_0 at rest located beyond a gravitational field and we move it to increase its potential energy by performing work in a gravitational field, causing an index of refraction n, this same mass will become equal to m':

$$m' = \frac{m_0}{n} \qquad \textbf{(30)}$$

Unfortunately we do not have scales accurate enough to verify the accuracy of this phenomenon, but like nuclear fusion, the mass decreases when subjected to a gravitational field.

When we drop a mass on Earth, the mass in question somehow creates a gravitational link (like a chemical or nuclear link) with Earth. The creation of this link is exothermic and releases energy, therefore a mass (according to Einstein's equation $E = m_0 \cdot c^2$).

On the contrary, if we raise a mass at a certain height, we perform work to give it some potential energy. This work, to increase the potential energy of the object, is given to the mass by the absorption of photons. The principle of conservation of energy is therefore preserved.

We have ignored, until now, the influence of approaching m_2 towards m_1. In the original equation, that is to say, in the Schwarzschild equation **(5)** we always assumed that m_1 was much larger than m_2 and that the total mass variation was done in m_2. However, even if the influence of m_2 on m_1 is small (and can often be neglected in the calculations), it is there.

4. VARIATION OF THE DIMENSIONS AND OF THE TIME OF OBJECTS AS A FUNCTION OF THE INDEX OF REFRACTION OF THE VACUUM

By analogy with relativity, we want to show here that in addition to changing the speed of light in vacuum (according to equation **(5)**) and the mass m_0 at rest of objects, the index of refraction in the vacuum also changes the length l_0 at rest and the time t_0 of objects at rest.

The reader will note that we make a parallel between the index of refraction n and the Lorentz factor using the following equation, as if the fact that an animated object with a velocity v corresponds to increasing locally the index of refraction n of the vacuum.

$$n \Leftrightarrow \sqrt{1-\frac{v^2}{c^2}} \tag{31}$$

In the following table, the special relativity equation [2,3] is found to the left. To the right, we find the equation that is associated to the index of refraction n of the vacuum:

ACCORDING TO THE SPECIAL RELATIVITY	INFLUENCE OF THE INDEX OF REFRACTION OF THE VACUUM	
We know that $m' = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$	So $m' = \frac{m_0}{n}$	**(32)**
We know that $t' = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$	So $t' = \frac{t_0}{n}$	**(33)**
We know that $l' = l_0\sqrt{1-\frac{v^2}{c^2}}$	So $l' = l_0 \cdot n$	**(34)**

All these effects are relativistic. In our calculations, we assume an observer located at infinity, which, ultimately, does not exist. However, even if the observer was located at the edge of the observable universe, that is to say at R_u, he could observe the same results that we have calculated. In practice, only the differences between two points located within our universe are of interest. So we just make the difference between the characteristics of objects between these two points. It is not necessary to have an observer located outside of our universe (to infinity).

Let's try to visualize the implications of these equations in a relatively simple experiment. Let's suppose an observer located on top of a mountain 1,000 meters above the sea level. For calculation purposes, we assume that the radius of the Earth is exactly $r_t = 6378137$ m and the Earth has a mass exactly equal to $m_t = 5.9736\times10^{24}$ kg. He has with him two rulers each measuring precisely each

1 m long, each with a mass of 1 kg. Also suppose that he has two infinitely precise clocks that beat both at the same rate of one second every second. A colleague moves one of the rules and one of the clocks at sea level. He keeps the other rule and the other clock to use it, for comparison, as a standard of measurement. Let's suppose that it is possible to observe the characteristics of the rule and of the clock from the point of observation located on top of the mountain. In reality, it is very difficult to make these remote observations (even impossible). Do not forget that we can not use a digital scale that moves with the object under observation as this would move the standard at the sea level and the observed difference would be zero.

If the whole experience is made within the rules, the observer will see that the characteristics of the ruler and the clock have changed from the values that can be measured on the ruler and the clock standards. He will find that the rule now located at sea level has a lower mass of about 2.18×10^{-13} kg, as time passes faster 2.18×10^{-13} s/s and the length is greater than by about 2.18×10^{-13} m. The effect is minimal and quite negligible on our scale.

In this experiment, the index of refraction of the vacuum at the top of the mountain is n_0 and the one at sea level is n_1.

$$n_0 = \sqrt{\frac{1+\dfrac{2G\cdot m_t}{c^2\cdot(r_t+1000)}}{1-\dfrac{2G\cdot m_t}{c^2\cdot(r_t+1000)}}} \quad n_1 = \sqrt{\frac{1+\dfrac{2G\cdot m_t}{c^2\cdot r_t}}{1-\dfrac{2G\cdot m_t}{c^2\cdot r_t}}} \qquad \textbf{(35)}$$

So the mass difference observed between the observer located at the top of the mountain and the one located at sea level will be:

$$\left(\frac{1}{n_1}-\frac{1}{n_0}\right)\cdot 1\,\text{kg} \approx -2.18\times10^{-13}\text{kg} \qquad \textbf{(36)}$$

For each second, the time difference between the clocks will be:

$$\left(\frac{1}{n_1}-\frac{1}{n_0}\right)\cdot 1\,\text{s} \approx -2.18\times10^{-13}\text{s} \qquad \textbf{(37)}$$

The difference in length will be:

$$\left(n_1-n_0\right)\cdot 1\,\text{m} \approx 2.18\times10^{-13}\text{m} \qquad \textbf{(38)}$$

We note that the effect is relativistic. Therefore, if the observer comes down the mountain with his standards along his collaborator, the differences between the characteristics of the standards and the objects under observation will be null.

5. CONCLUSION

Using one of the Schwarzschild equations giving the index of refraction of the vacuum, we have shown that the variation of the potential energy of an object can be directly associated with a variation of the refractive index of the vacuum. Similarly, we have shown that in addition to affecting the speed of light in the vacuum, the refractive index of the vacuum has a direct influence on the masses, dimensions and time values of the objects at rest. Of course, we must add the relativistic influence of the speed of the objects to the influence of the refractive index of the vacuum.

Knowing that the masses, the time and the space dimensions may depend on the conditions of the gravitational fields in which these observations are made, we may be better able to understand and predict the phenomena observed in the presence of more intense gravitational fields such as around black holes.

6. REFERENCES

[1] "Latest (2010) Values of the Constants", NIST Standard Reference Database 121, latest update: April 2012, paper available on Internet at: http://physics.nist.gov/cuu/Constants/index.html

[2] Einstein, Albert, “La relativité”, *Petite Bibliothèque Payot*, v. 25, Parish, original 1956 edition from Gauthier-Villar fully reused by the Payot & Rivages editions for the 2001 edition, p. 109.

[3] Einstein, Albert, "On the Electrodynamics of Moving Bodies ", *The Principle of Relativity (Dover Books on Physics)*, New York, Dover Publications, 1952 (original 1905 paper), pp. 35-65.

[4] Mercier, Claude, "The Speed of Light May not be Constant", *Pragtec*, Baie-Comeau, Quebec, Canada, October 8, 2011, paper available on Internet at: www.pragtec.com/physique/

[5] Binney, James and Michael Merrifield, “Galactic astronomy”, *Princeton University Press*, 1998, p. 733, from the equation A2.

[6] Maneghetti, Massimo, "Introduction to Gravitational Lensing, Lecture scripts", *Institut für Theoretische Astrophysik*, Bologna, Italy, 2006, p. 7, from equation 1.19, Web. <http://www.ita.uni-heidelberg.de/~massimo/sub/Lectures/chapter1.pdf>

[7] Hubble, E. & Humason, M. L., "The Velocity-Distance Relation among Extra-Galactic Nebulae", *The Astrophysical Journal*, v. 74, 1931, p.43.

[8] Einstein, Albert, "On the Influence of Gravitation on the Propagation of Light", *The Principle of Relativity (Dover Books on Physics)*, New York, Dover Publications, 1952 (original 1911 paper), pp. 97-108.

[9] Mercier, Claude, "Calculation of the Universal Gravitational Constant *G*", *Pragtec*, Baie-Comeau, Quebec, Canada, March 13, 2013, paper available on Internet at: www.pragtec.com/physique/

[10] Maneghetti, Massimo, "Introduction to Gravitational Lensing, Lecture scripts", *Institut für Theoretische Astrophysik*, Bologna, Italy, 2006, p. 7, from equation 1.19, Web. <http://www.ita.uni-heidelberg.de/~massimo/sub/Lectures/chapter1.pdf>

[11] Matzner, Richard A., "Dictionary of Geophysics, Astrophysics, and Astronomy (Comprehensive Dictionary of Physics)", *Boca Raton: CRC*, 2001.

[12] Mercier, Claude, "Calculation of the Apparent Radius of Curvature of the Universe", *Pragtec*, Baie-Comeau, Quebec, Canada, June 9, 2013, paper available on Internet at: www.pragtec.com/physique/

[13] Wang, Xiaofeng et al., "Determination of the Hubble Constant, the Intrinsic Scatter of Luminosities of Type Ia SNe, and Evidence for Non-Standard Dust in Other Galaxies", March 2011, pp. 1-40, arXiv:astro-ph/0603392v3

[14] Mercier, Claude, "Variation of the « Physics Constants » Over Time", *Pragtec*, Baie-Comeau, Quebec, Canada, March 16, 2011, paper available on Internet at: www.pragtec.com/physique/