Calculation of the Apparent Mass of the Universe

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Einstein showed that an imposing mass could bend space-time and change the index of refraction of the vacuum surrounding the mass[1]. *This principle is also the basis of gravitational lenses* [5,7].

The biggest existing mass is the universe itself. Einstein showed that the universe is finite [13]. *He thought it was static* [13], *but Edwin Powell Hubble showed in 1929, by the observation of far galaxies, that the universe is in expansion* [14].

Assuming that the universe, on a large scale, consists of material arranged uniformly [13], *we can do the hypothesis that the mass of the universe will influence the speed of light over the time* [9]. *Only a specific given mass m_u has the capability to bend the space-time of the universe such a way to influence the speed of light so that it has the value c for the current apparent radius of curvature. With only the parameters c (actual speed of light in the vacuum), G (universal gravitational constant) and H_0 (Hubble constant), it is possible to calculate precisely the apparent mass of the universe.*

According to our calculations, the apparent mass of the universe would be about 1.8×10^{53} kg. [2]

KEY WORDS: Mass of the universe, Planck time, Planck mass, Einstein, Hubble

1. INTRODUCTION

The current space-time curvature of the universe is a function of three parameters: c (the speed of light in the vacuum), G (the constant of universal gravitation) and H_0 (the Hubble constant) [9]. These parameters are imposed by the mass of the universe and its current apparent curvature radius. Therefore, whatever the constitution of the universe is (matter, energy, light, dark matter or

[1] His first paper on the subject dates from 1911 [3]. However, using the general relativity bases [4], he had to multiply the index of refraction variation by a factor 2 [7,8]. His theory is confirmed by the gravitational lenses [5,7].

[2] Recent works [19] on the Hubble constant H_0 and on the universal gravitational constant G allowed to revaluate the apparent mass of the universe to $m_u \approx 1.728098528(14)\times10^{53}$ kg as a function of the CODATA 2014 [20].

other), it is possible to calculate what should be the mass of the universe in order to create the current space-time curvature. This is the reason why we call this, the "apparent" mass of the universe.

We want to show that it is possible to calculate the apparent mass of the universe in different ways and these methods lead to the same result. It will be calculated as a function of only the following parameters *c*, *G* and H_0.

We will begin to calculate the apparent mass of the universe by using the momentum conservation principle. We will finish by a method using Planck units.

The proprieties of actual matter are intimately related to the apparent mass of the universe. That is why it is useful to know it.

2. DEVELOPPEMENT

2.1. Calculation of the Apparent Mass of the Universe Using the Momentum Conservation Principle

Let's calculate the apparent mass of the universe from the momentum conservation principle. It is applied to the apparent mass of the universe, at the position of the apparent radius of curvature the luminous universe's envelope.

The momentum conservation principle says that a force *F* applied during a delay Δt will move a mass *m* by increasing its velocity by Δv.

$$F \cdot \Delta t = m \cdot \Delta v \tag{1}$$

The apparent age of the universe [10] is given by:

$$\Delta t = \frac{1}{H_0} \tag{2}$$

The value of H_0 is probably situated between 70.4 [11] and 76.9 km/(s·MParsec) [6]. In this present document, we will use the value of 70.4 km/(s·MParsec) since it comes from the most recent results of the NASA's WMAP project [11]. [3]

The expansion of the material universe and the expansion of the luminous

[3] Recent works allowed to evaluate precisely the value of the Hubble constant to $H_0 \approx 72.09548580(32)$ km/(s·MParsec) [18].

universe began from a zero speed at the horizon [9]. The material universe is expanding at a lower rhythm than the luminous universe [9]. Currently, the expansion of the luminous universe is at the speed of light c [2]. In equation (1), luminous universe's envelope undergoes a variation of speed $\Delta v = c$ over time.

Let's suppose that we take a container with walls infinitely lightweight, strong and elastic. We place a mass m inside it and by any means we make it disintegrate into pure energy (in photons). The total mass associated to the energy of the light within the container will then be equal to m. Therefore, we are forced to associate a mass to the photons, even if it seems to be infinitely small. We will therefore designate this mass as being the mass-energy.

Let's note here that the mass-energy of the luminous universe is in expansion at the speed of light ($c \approx 299792458$ m/s). This is a mass associated to the luminous energy by the Einstein equation:

$$E = m_0 \cdot c^2 \tag{3}$$

Since the mass-energy already moves to the speed of light, we should not try to apply the Lorentz factor on it. If it was, we would encounter incongruities.

The material universe is located inside the luminous universe's sphere. Matter moves at about $\beta \approx 0.76{\cdot}c$. According to NASA, the mass of the matter that constitute our universe does not represent more than 4.49 % of the total mass m_u of it [11]. Therefore, it has very little impact on our calculation. We must understand that even if 1 m^3 of vacuum represents a very small mass, there is a huge amount of vacuum in the universe. It is like multiplying an infinitely small number by an infinitely huge one. The result is not necessarily zero.

Considering that the apparent mass of the actual universe is m_u (including any kind of mass of energy), we get the following momentum:

$$F \cdot \Delta t = m_u \cdot c \tag{4}$$

Knowing that the speeds involved in this process are very large, we use the relativistic mass of the universe. In equation (4), we assume that the universe consists mainly of mass-energy capable of moving at speeds close to light. It is especially true for the luminous universe that is made of light.

Newton's law predicts the gravitational attraction force F exerted between a mass m_1 and a mass m_2 separated by a distance r. G represents the universal gravitational constant and is equal to $G \approx 6.67284(80) \times 10^{-11}$ m^3/(kg·s^2) according to the CODATA 2010 [19].

$$F = \frac{G \cdot m_1 \cdot m_2}{r^2} \tag{5}$$

In fact, only the centers of masses are used to determine the distances between the masses. If we apply this principle to the universe as a whole, we can calculate the force exerted, by the mass of the universe m_u on itself, so that the universe is contracting. This force will be equal to the universe's force of expansion. R_u represents the luminous universe's apparent radius of curvature and m_u represents the apparent mass of the universe.

$$F = \frac{G \cdot m_u^2}{R_u^2} \tag{6}$$

The luminous universe expands at the speed of light [2]. Therefore, the universe's apparent radius of curvature, that is to say, the luminous world, is:

$$R_u = c \cdot \Delta t = \frac{c}{H_0} \tag{7}$$

Of course, for our current position, that is to say in the material universe, the masses (such as stars, planets, galaxies, etc.) cannot move at the speed of light. The material universe is therefore in expansion inside the luminous universe. Let's suppose that the material universe's apparent radius of curvature at our location is r_u. The material universe is in expansion at a speed equal to $\beta \cdot c$ [3].

$$\beta = 3 - \sqrt{5} \approx 0.76 \tag{8}$$

Consequently:

$$r_u = \frac{\beta \cdot c}{H_0} \text{ et } R_u = \frac{r_u}{\beta} \tag{9}$$

By using the equations (2), (4), and (6), by simplifying and rearranging the terms, we obtain:

$$m_u = \frac{c \cdot R_u^2 \cdot H_0}{G} = \frac{c \cdot r_u^2 \cdot H_0}{G \cdot \beta^2} \tag{10}$$

By using the equation (9), equation (10) becomes:

$$m_u = \frac{c^3}{G \cdot H_0} \approx 1.8 \times 10^{53} \text{kg} \tag{11}$$

This result is similar to the proportionality relation found by Mr. Carvalho [1]:

$$m_u \propto \frac{c^3}{G \cdot H_0} \tag{12}$$

By omitting the fact that a constant of proportionality is missing, our equation is similar to that of Mr. Carvalho.

2.2. Calculation of the Apparent Mass of the Universe from Planck Units

Let's calculate the apparent mass of the universe from the concept of the Planck units. Planck units come mainly from the Planck time definition which comes from the Heisenberg uncertainty principle [12].

We begin by associating a mass to the photon which has as wavelength, the radius of the universe. Then, we will show that the Planck unit is not an arbitrary unit of time, but it is the smallest unit of time that exists. From there, we will find the Planck length and the Planck mass. From these Planck units, we will find a proportionality relationship which will allow for finding the mass of the universe.

2.2.1. Basic Hypothesis on Matter

We understand that the vacuum (absence of molecules and atoms) is not nothingness (total absence of anything). The term "vacuum" unfortunately refers to a concept of absence of matter. Moreover, it is conceivable that the matter as we understand it now (with atoms and molecules) does not represent all existing forms of matter. While astrophysicists are convinced of the existence of dark matter, they have unfortunately never managed to get hold of what it really represents.

According to our conception of the vacuum, it would actually be a bath of photons of different wavelengths. These photons exert a pressure of radiation on matter. Tor this reason, the word "ether", which is the former name of the vacuum, seems more appropriate. Of course, the density of the ether would be extremely low, but in a sufficiently large volume, the mass of the ether becomes significant. Matter as we know it represents, according to NASA, approximately 4.49 % of the total mass of the universe (the rest being dark matter with 22.2 % and dark energy with 73.4 %) [11].

Although the purpose of our study is not to develop a final concept of what matter is, we hypothesize that all matter and ether (the vacuum), are made of photons. In matter, the photons would combine with others to form particles (electrons, protons, neutrons, etc.).

The photon is the smallest known quantity of matter. It is so small that we have long believed that its mass was zero.

In 1900, Planck showed that the energy of a photon is given by:

$$E = h \cdot f = \frac{h \cdot c}{\lambda} \tag{13}$$

In 1905, Einstein established that the energy of a body at rest is given by the following equation [15]:

$$E = m_0 \cdot c^2 \tag{14}$$

In 1924, De Broglie wanted to highlight the duality between wave and particle [16,17] by making equal the equations (13) and (14). The new resulting equation associates a photon to a mass. Here, we replace m_0 by m_{ph} to designate the mass associated to the photon.

$$m_{ph} \cdot c^2 = h \cdot f \tag{15}$$

For a particle with radius r turning at the speed of light c, the frequency f is given by:

$$f = \frac{c}{2 \cdot \pi \cdot r} \tag{16}$$

Of course, when we isolate m_0, we get the mass associated to the photon at rest with respect to us (we mean with respect with the matter in expansion in the universe):

$$m_{ph} = \frac{h \cdot f}{c^2} \tag{17}$$

If the wavelength of this photon was precisely equal to the apparent radius of the luminous universe (see equation **(7)**), we would get:

$$f = \frac{c}{2 \cdot \pi \cdot R_u} = \frac{H_0}{2 \cdot \pi} \tag{18}$$

By using equation (18), equation (17) can be rewritten as following:

$$m_{ph} = \frac{h \cdot H_0}{2 \cdot \pi \cdot c^2} \approx 2.7 \times 10^{-69} \text{kg} \tag{19}$$

Let’s notice that the mass associated to the photon is extremely small. It is undetectable. That is the reason for which it has never been measured.

2.2.2. Planck Time

Planck time is a time unit called "natural" because it depends only on known constants such as the gravitational constant G, Planck's constant h and the speed of light c. Planck time is not only a time measurement unit. Unlike conventional time units (e.g. seconds), Planck time is not chosen arbitrarily. It has an intrinsic physical meaning.

In this document, we want to show where this unit comes from and most importantly, what it really means. To do this, we will first show where the Plank time equation comes from. Then, we will calculate the mass of the universe with it.

Heisenberg enunciated for the first time in 1927, the uncertainty principle which is now on of the foundations of quantum mechanics [12]. This principle says that it is not possible to know precisely the speed and the position of an object simultaneously.

In a second statement, Heisenberg teaches us that the uncertainty in the measurement of a body's energy is inversely proportional to the duration of the measure. Another way to formulate this statement is to say that the product of the energy ΔE by the time Δt must be:

$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2} \tag{20}$$

Here, the Planck constant $\hbar = h / 2 \cdot \pi$ and h = 6.62606957(29) x 10^{-34} J·s according to the CODATA 2010 [18].

For matter at rest, the energy is:

$$\Delta E = m_0 \cdot c^2 \tag{21}$$

Considering only the case where Δt is the smallest possible quantity of time, we rename Δt by t_p and we only keep the equality in (20). Then, isolating t_p, we get:

$$t_p = \frac{\hbar}{2 \cdot m_0 \cdot c^2} \tag{22}$$

Now, let's suppose that we take a photon at rest with a mass m_0, which is at the position of infinity, and let's suppose that we confine this photon in a sphere of radius r. The variation of potential energy ΔE_p spent would be:

$$\Delta E_p = -G \cdot {m_0}^2 \cdot \left(\frac{1}{r_\infty} - \frac{1}{r} \right) \Bigg|_{r_\infty = \infty} = \frac{G \cdot {m_0}^2}{r} \tag{23}$$

By choosing r so that the variation of potential energy ΔE_p corresponds to half ΔE, we obtain the following special case:

$$\frac{m_0 \cdot c^2}{2} = \frac{G \cdot m_0{}^2}{r} \tag{24}$$

If we isolate r, we obtain exactly what we call the Schwarzschild radius:

$$r = \frac{2 \cdot G \cdot m_0}{c^2} \tag{25}$$

This is the radius of a black hole [5]. For this radius, the speed of light becomes zero. Isolating m_0, we obtain:

$$m_0 \cdot = \frac{r \cdot c^2}{2 \cdot G} \tag{26}$$

By using (26) in (22), we obtain:

$$t_p = \frac{\hbar \cdot G}{r \cdot c^4} \tag{27}$$

But we can obtain r from the following relation:

$$r = c \cdot t_p \tag{28}$$

Therefore, equation (27) becomes:

$$t_p = \sqrt{\frac{\hbar \cdot G}{c^5}} = \sqrt{\frac{h \cdot G}{2 \cdot \pi \cdot c^5}} \approx 5.4 \times 10^{-44}\,\mathrm{s} \tag{29}$$

This equation allows one to calculate the smallest measurable unity of time [5]. It is known as « Planck time».

In his work on relativity, Einstein postulated that nothing can travel faster than light [4,15]. Therefore, Planck length [5] could be defined as being the greatest distance covered during the smallest unit of time. Planck time t_p must be defined like this:

$$L_p = c \cdot t_p = \sqrt{\frac{h \cdot G}{2 \cdot \pi \cdot c^3}} \approx 1.6 \times 10^{-35}\,\mathrm{m} \tag{30}$$

Let's associate a mass m_p to a particle of radius L_p which would rotate on itself at the speed of light. Let's note that when the wavelength is smaller, it is more energetic. As the particle we are currently interested in has the smallest wavelength, it becomes the particle which has the highest energy level that exists. By using equations similar to equation (15) and (16), we get:

$$m_p \cdot c^2 = \frac{h \cdot c}{2 \cdot \pi \cdot L_p} \tag{31}$$

By using the equation (30), and isolating m_p from equation (31), we get the equation which defines Planck mass [5]:

$$m_p = \sqrt{\frac{h \cdot c}{2 \cdot \pi \cdot G}} \approx 2.2 \times 10^{-8}\text{kg} \tag{32}$$

Planck units are currently used in physics. They all represent well defined limits. Therefore, these are not units taken at random.

2.2.3. Proportions of the Universe Identical to the Planck Particle

Here, we will show that from Planck units, we are able to show that the mass/radius ratio of the universe is the same than the masse/Planck length. By putting this game proportionality in evidence, it becomes easy to calculate the mass of the universe.

By using equation (29) and by doing a few manipulations, we get:

$$\frac{2 \cdot \pi \cdot \mathrm{c}^5}{h \cdot G} = \frac{1}{t_p^2} \tag{33}$$

In showing the Hubble constant H_0 on each side of the equation and rearranging it a bit to highlight certain terms, we get:

$$\frac{\left(\frac{c^3}{G \cdot H_0}\right)}{\left(\frac{c}{H_0}\right)} = \frac{h}{2 \cdot \pi \cdot c^3 \cdot t_p^2} \tag{34}$$

On the basis of the Planck time definition of equation (29), we get:

$$\frac{\left(\frac{c^3}{G \cdot H_0}\right)}{\left(\frac{c}{H_0}\right)} = \frac{\sqrt{\frac{h \cdot c}{2 \cdot \pi \cdot G}}}{\sqrt{\frac{h \cdot G}{2 \cdot \pi \cdot c^3}}} = \frac{c^2}{G} \tag{35}$$

This equation can be rewritten as follows:

$$\frac{m_u}{R_u} = \frac{m_p}{L_p} = \frac{c^2}{G} \tag{36}$$

In this last equation, m_u is the apparent mass of the universe, R_u is the apparent radius of the luminous universe, m_p is the Planck mass and L_p is the Planck length.

According to this equation, the mass/radius proportions of the universe are the same as the Planck particle.

In addition to the proportional relationship shown in equation (36), note that the following equalities are also verifiable and usable:

$$\frac{m_u}{R_u} = \frac{m_p}{L_p} = \frac{m_p^2}{m_e \cdot r_{ce}} = \frac{m_p^2 \cdot \alpha}{m_e \cdot r_e} = \frac{m_p^2}{m_{pr} \cdot r_{cpr}} = \frac{m_p^2}{m_{ph} \cdot \lambda} = \frac{m_p^2}{m_x \cdot \lambda_x} = \frac{m_p^2 \cdot \alpha}{e^2 \cdot 10^{-7}} = \frac{c^2}{G} \tag{37}$$

In this equation, m_e represents the mass of the electron, r_{ce} is the Compton radius of the electron, r_e is the classical radius of the electron, α is the fine structure constant, m_{pr} is the mass of the proton, r_{cpr} is the Compton radius of the proton, e is the charge of the electron and m_{ph} is the mass coming from the energy associated to a photon having the wavelength λ. In fact, in a general manner, every elementary particle having a mass m_x less than the Planck mass (if not, it would not be elementary and would rather be a cluster of particles), it is possible to use the wavelength λ_x associated to this particle to verify the above equalities.

Although these equations are interesting, demonstrating these would be tedious and would not contribute anything to this document. They are therefore left to the reader.

2.3. Calculation of the Apparent Mass of the Universe from Planck Mass

Let's calculate the apparent mass of the universe m_u from a comparison with Planck mass m_p.

By using equations (30), (32) and (36), we obtain the apparent mass of the universe m_u:

$$m_u = \frac{c^3}{G \cdot H_0} \approx 1.8 \times 10^{53} \text{kg} \tag{38}$$

This equation is the same than equation (11).

2.4. Calculation of the Apparent Mass of the Universe from the Energy of a Photon (added May 4rd 2019)

Here, we want to calculate the apparent mass of the universe m_u by equating the gravitational energy and of a photon with the mass energy contained in a photon (when looked as being a corpuscle).

Let us define R_u as being the apparent radius of curvature of the luminous universe. It is given by the following equation:

$$R_u = \frac{c}{H_0} \tag{39}$$

Let us associate a mass m_{ph} with a photon that is located at the periphery of the luminous universe (with a wavelength of $2\pi R_u$).

$$m_{ph} = \frac{h}{2\pi R_u \cdot c} \tag{40}$$

If we place this photon at the periphery of the luminous universe, at a distance R_u from the center of mass of the universe, it will have an E_g gravitational energy.

$$E_g = \frac{G m_u m_{ph}}{R_u} \tag{41}$$

According to the special relativity, the mass energy associated with this photon is E_m.

$$E_m = m_{ph} c^2 \tag{42}$$

By equating equations **(41)** and **(42)**, replacing R_u with equation **(39)**, and isolating m_u, we obtain the same equation as that of Carvalho [1].

$$m_u = \frac{c^2}{G \cdot R_u} = \frac{c^3}{G \cdot H_0} \tag{43}$$

2.5. Calculation of the Apparent Mass of the Universe from the Electron Mass (Added May 4th 2019)

In 2013, we discovered equations that give exactly the universal gravitational constant G and the Hubble constant H_0 as a function of the classical electron radius, the electron mass m_e, and the fine-structure constant α [19]. Using these equations as well as the values of CODATA 2014 [20], we obtain:

$$G = \frac{c^2 \cdot r_e \cdot \alpha^{20}}{m_e \cdot \beta} \approx 6.67229809(86)\ \text{m}^3 / (\text{kg} \cdot \text{s}^2) \tag{44}$$

$$H_0 = \frac{c \cdot \alpha^{19} \cdot \sqrt{\beta}}{r_e} \approx 72.09548580(32)\ \text{km} / (\text{s} \cdot \text{MParsec}) \tag{45}$$

We define β as the ratio between the expansion speed of the material universe and the expansion speed of the luminous universe (which is currently the speed of light c) [9].

$$\beta = 3 - \sqrt{5} \approx 0.76 \tag{46}$$

With equations (44) à (46), we modify equation **(43)** to get:

$$m_u = \frac{m_e \cdot \sqrt{\beta}}{\alpha^{39}} \approx 1.728098528(14)\ \text{kg} \tag{47}$$

This equation is the most precise of all.

3. CONCLUSION

In this work, we limited ourselves calculating the apparent mass of the universe in two ways: by the principle of conservation of momentum and by a principle of conservation of proportionality between the Planck particle and the universe.

Let's recall that we have already used the equation obtained for m_u in a previous work to calculate the acceleration of light [9]. As this acceleration causes the Pioneer effect and that our equation could successfully account for this phenomenon, we conclude that the equation used to calculate the mass of the universe is correct and verified by this phenomenon.

In the event of an error in the calculation of the mass of the universe, we would not have been able to predict the value of the Pioneer acceleration. It would not have matched so well with the NASA's observations (less than 2.5% of error) [9].

We suspect that the mass of the universe and the expansion of the latter are closely related to the structure of matter. A better understanding of the characteristics of our universe will perhaps, one day, allow for a better comprehension about the atom's structure and its properties.

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