

Calculation of the Age of the Universe

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Rev. May 4th, 2019

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Through his observations, Edwin Powell Hubble discovered in 1929 that the universe is expanding. He noticed that the galaxies, independently of their own movements, move away from each other at speeds even greater than when they were more distant from each other [1]. He deduced a law and a parameter that he called H_0 (better known as the Hubble constant).

Based on this fact, many astrophysicists approximate the extrapolated age of the universe using the equation $1/H_0$, especially if the density of the universe is low. This is the method used by NASA to calculate a universe age of 13.75 billion years (see the results of the seventh year of the WMAP project [3]).

However, the Friedman-Lemaître-Robertson-Walker model [4,5,6,7,8,9] predicts that the age of the universe would rather be around $2/(3 \cdot H_0)$.

Using the parameters c (the speed of light in the vacuum), G (the universal gravitational constant) and H_0 (the Hubble constant), it is possible to calculate precisely the age of the universe through an integral. This new method has the merit of reconciling the two methods mentioned above by making a fundamental distinction between the "real" and the "apparent" age of the universe.

KEY WORDS: Age of the universe, light, big bang, expansion of the universe

1. INTRODUCTION

The NASA currently estimates the age of the universe by using the inverse of the Hubble constant [3], that is to say $1/H_0$. The model presented by the physicists Alexandre Friedman, Monsignor George Lemaître, Howard P. Robertson and Arthur G. Walker predicts that for a flat universe dominated by the presence of matter [4,5,6,7,8,9], the true age of the universe would rather be around $2/(3 \cdot H_0)$.

Einstein was considering the universe as being static [18], with constant space-time dimensions. When he noticed that his general relativity theory was leading to a universe in expansion or in contraction, he added a cosmological constant to his equations to produce his model of the static universe [18]. Let's mention that in his theory of relativity, Einstein was taking for granted that the speed of light in the vacuum was constant [13]. It was entirely consistent with his view of the universe. A static universe leads to a constant speed of light, except, of course, on the outskirts of large masses [12,13].

In 1929, Hubble found that the universe was expanding [1]. When Einstein became aware of Hubble's observations, he was forced to admit that adding a cosmological constant to his model of the universe to make it static was the biggest blunder he has made in his life [18]. It looks like he did not see, at that moment, that the acceleration of light over time was a direct consequence of an expanding universe. However, with recent work, we showed that it is possible that the speed of light has never been constant over time [11].

According to Einstein, the gravitational potential (consequently, the mass) changes the index of refraction of the vacuum and slows the light down.¹ The universe is expanding [3] and we move away from its centre of mass [6]. Therefore, the index of refraction diminishes and light accelerates over time [11].

Initially, we will first show how the approximate age of the universe is calculated. In a second step, we will use some results from a work we have done recently [11] to estimate the age of the universe by performing the integral of the inverse of the expansion velocity of the material universe according to the curvature radius of the universe. Finally, we will approximate the age of the universe. We will show that $1/H_0$ actually represents a good approximation of the apparent age of the universe and that $2/(3 \cdot H_0)$ represents the real part of the age of the universe. We can then compare the results and comment.

2. DEVELOPMENT

2.1. Current Methods for the Calculation of the Age of the Universe

In 1929, Edwin Powell Hubble found that galaxies distance themselves from one another at a speed proportional to the distance between them [1]. He deduced a law involving a constant that he called H_0 . It represents the average recession velocity v of galaxies per unit of distance Δr .

$$\frac{v}{\Delta r} = H_0 \quad \text{where} \quad v = \frac{\Delta r}{\Delta t} \quad (1)$$

The value of H_0 is probably between 70.4 [3] and 76.9 km/(s·MParsec) [17]. In this paper, we will use the value of 70.4 km/(s·MParsec), since it comes from the most recent results of the WMAP project of NASA [3]. From the equation (1), we get:

¹ His first paper on the subject dates from 1911 [12]. However, using the general relativity bases [13], he had to multiply the index of refraction variation by a factor 2 [14,15]. His theory is confirmed by the gravitational lenses [15,16].

$$\Delta t = \frac{1}{H_0} \quad (2)$$

It is with this equation that NASA is currently evaluating the age of the universe in the approximation of 13.75 billion years [3].² This way of calculating the age of the universe assumes that the expansion rate of the latter is constant.

According to the model of Friedman-Lemaître-Robertson-Walker [4,5,6,7,8,9], the true age of the universe would rather be around:

$$\Delta t = \frac{2}{3 \cdot H_0} \approx 9.3 \times 10^9 \text{ years} \quad \text{or} \quad \Delta t \cdot H_0 = \frac{2}{3} \quad (3)$$

2.2. Calculation of the Age of the Universe

A recent work that we made leads us to believe that the speed of the light and the speed of expansion of the material universe have not been constant over time [11]. We will use some results from this work to solve the integral of the inverse of the speed of expansion of the material universe as a function of the radius of curvature of the universe. In this way, we will calculate the age of the universe.

Some Einstein works showed that the presence of a massive body changes the shape of the space-time [13] and increases the index of refraction of the vacuum around the body [12]. By moving away from the body, the gravitational influence is being reduced and the speed of light tends toward c .

To summarize our previous work [11], we can say that the universe itself is the biggest existing mass. Since the universe is expanding [1], we move away from the center of mass of the latter. This causes the index of refraction to diminish over time. The speed of light is therefore increasing over time to eventually tend toward a constant that we named k (when the apparent radius of curvature of the universe tends toward the infinity). The current speed of light in the vacuum is c . According to our previous work [11], the speed of light increases by 1 m/s every 35.4 years (the acceleration of light is about $a_L = c \cdot H_0 / \beta \approx 8.95 \times 10^{-10} \text{ m/s}^2$).

² Recent works allow us to evaluate precisely the value of the Hubble constant to 72.09548580(32) km/(s·MParsec) [19]. Please refer to Appendix A for more details.

We summarize here the values of the various parameters of the universe which were calculated in the work cited in [11]. These parameters were based only on c (the speed of light in vacuum), G (the universal gravitational constant) and H_0 (the current Hubble constant).

The ratio β between the current speed v_m of the expansion of the material universe and the current speed of light c is [11]:

$$\beta = 3 - \sqrt{5} \approx 0.76 \quad (4)$$

The asymptotical speed of light k (when the apparent radius of curvature of the universe will tend towards infinity) is [11]:

$$k = c \cdot \sqrt{2 + \sqrt{5}} \approx 2 \cdot c \approx 6 \times 10^8 \text{ m/s} \quad (5)$$

The apparent mass m_u of the universe is [10,11] :

$$m_u = \frac{c^3}{G \cdot H_0} \approx 1.8 \times 10^{53} \text{ kg} \quad (6)$$

The apparent radius of curvature r_u of the universe is [11]:

$$r_u = \frac{\beta \cdot c}{H_0} \approx 1.0 \times 10^{26} \text{ m} \quad (7)$$

As we mentioned previously, Einstein showed that the speed of light is influenced by the gravitational potential. According Schwarzschild's calculations based on the general relativity, it is possible to calculate the speed of light v_c as a function of r [15,16].

$$v_c(r) = \frac{c}{n(r)} \quad \text{where} \quad n(r) = \sqrt{\frac{1+\omega}{1-\omega}} \quad \text{and} \quad \omega = \frac{2 \cdot G \cdot m}{c^2 \cdot r} \quad (8)$$

$v_c(r)$ = New speed of light as a function of the distance r
 R = Distance between the centre of mass m and where $v_c(r)$ is evaluated
 M = Mass creating the gravitational field
 G = $6.673 \ 84 \times 10^{-11} \text{ m}^3/(\text{kg} \cdot \text{s}^2)$ = Universal gravitational constant
 C = $2.997 \ 924 \ 58 \times 10^8 \text{ m/s}$ = Speed of light in vacuum

This equation is valid for the calculation of the speed of light around a black hole, a star or a galaxy. Another way of expressing this: this equation is only valid for the present time.

However, as soon as we try to calculate the speed of light for the time in the past or in the future, we must take into account the fact that the speed of light changes

as a function of the apparent radius of curvature of the universe [11]. We must therefore change c in the equation (8) to k (see equation (5)). The speed of light as a function of the apparent radius of curvature becomes:

$$v_L(r) = \frac{k}{n(r)} \text{ where } n(r) = \sqrt{\frac{1+y}{1-y}} \text{ and } y = \frac{2 \cdot G \cdot m_u}{k^2 \cdot r} \quad (9)$$

In this equation, there is a radius of curvature r_h for which the speed of light $v_L(r_h) = 0$. This position r_h is called the horizon of the universe. This is the position for which the denominator of the square root of the equation (9) becomes zero. In a similar way, in a black hole, the radius of curvature of the horizon r_h is obtained by the Schwarzschild's radius where we replace c by k :

$$r_h = \frac{2 \cdot G \cdot m_u}{k^2} \approx 6.2 \times 10^{25} m \quad (10)$$

It is the same principle as that of a black hole [11]. In fact, the universe is the biggest existing black hole since it has the biggest mass. However, unlike a black hole, rather than having the mass located within the limits of the horizon, a big part of the black hole mass of the universe lies outside the boundaries the horizon. In fact, the black hole of the universe is located around the center of the mass of the universe.

The speed of the expansion of the universe is currently the speed of light [2]. Based on the principles of Einstein's relativity, matter cannot move at the speed of light without having an infinite energy. Consequently, the previous assertion can be true only for light. The material universe (containing the galaxies, intergalactic dust clouds, etc.) is expanding at a speed equal to $\beta \cdot c$. The factor β must necessarily be less than 1. Moreover, according to our equation (4), its value would be around 0.76.

When the universe began its expansion from $r = r_h$, the expansion started with a speed equal to zero (since the speed of light is zero for this radius of curvature).

For a radius of curvature r greater than r_h , the speed of the expanding material universe $v_m(r)$ is always β times the speed of light $v_L(r)$:

$$v_m(r) = \beta \cdot v_L(r) = \frac{\beta \cdot k}{n(r)} \text{ where } n(r) = \sqrt{\frac{1+y}{1-y}} \text{ and } y = \frac{2 \cdot G \cdot m_u}{k^2 \cdot r} \quad (11)$$

Performing the integral of the inverse of the expansion velocity of the material universe $v_m(r)$ relative to the radius of curvature r , it is possible to calculate the age of the universe more precisely than by using a single tangential projection.

Let's find the age of the universe T_u by performing the following integral between the radius of curvature of the horizon r_h and the apparent radius of curvature of the universe r_u :

$$T_u = \int_0^{r_u} \frac{1}{v_m(r)} \cdot dr = T_{0h} + T_{hu} = \int_{r_h}^{r_u} \frac{1}{v_m(r)} \cdot dr + \int_0^{r_h} \frac{1}{v_m(r)} \cdot dr \quad (12)$$

Performing the integral calculation, we obtain:

$$\int \frac{1}{v_m(r)} \cdot dr = \frac{(z(r) + 2 \cdot G \cdot m_u \cdot \ln(2 \cdot [k^2 \cdot r + z(r)]))}{\beta \cdot k^3} \quad (13)$$

$$\text{where } z(r) = \sqrt{k^4 \cdot r^2 - 4 \cdot G^2 \cdot m_u^2}$$

Consequently, the value of T_u becomes:

$$T_u = T_{0h} + T_{hu} \approx (9.73 + 10.72 \cdot i) \times 10^9 \text{ years where } i = \sqrt{-1} \quad (14)$$

This result is of a complex type. In the equation (14), the first part of the integral is of a real type (between r_h and r_u). However, the second part of this latter one is of an imaginary type (between 0 and r_h).

For now, no one is able to observe what happens inside the limits of the horizon of a black hole (between 0 and r_h). In this paper, we make no conjecture in regard to what goes on inside a black hole. The physical interpretation of a complex type of time is left to posterity for analysis. What is certain is that only the portion of the time elapsed T_{hu} (around 9.73 billion years) between the radius of curvature of the horizon r_h (where the speed of light becomes zero) and the apparent radius of curvature of the universe r_u is of a real type and has good and well elapsed. When we wish to consider the time elapsed between the position 0 of the big bang and the radius of curvature of the horizon r_h , we must calculate the module of the time elapsed $|T_u|$. We define this value as the apparent age of the universe because it does not necessarily represent the true age of the universe. This number represents only an apparent age in the likely event that the big bang existed.

$$|T_u| = |T_{0h} + T_{hu}| = \sqrt{T_{0h}^2 + T_{hu}^2} \approx 14.48 \times 10^9 \text{ years} \quad (15)$$

We see that the obtained value is only 4.3 % over the estimated value of (2).

3. APPROXIMATION OF THE AGE OF THE UNIVERSE

As in the calculation of the power for an electrical motor (with the real power, the inductive power and the apparent power), the age of the universe may be seen as follows: the « real » part of the age of the universe, the « imaginary » part of the

age of the universe and the « apparent » age of the universe. The module of the two components (real and imaginary) can be calculated using the Pythagorean theorem by finding the square root of the sum of the squares of the real part of the age of the universe and the imaginary part of the age of the universe.

The calculation of the approximation of the age of the universe will be made in three parts: the approximation of the real part of the age of the universe, the approximation of the imaginary part of the age of the universe and the calculation of the module of the apparent age of the universe.

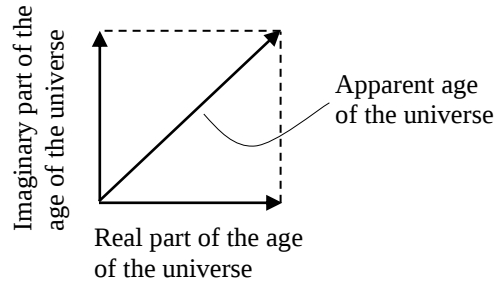


Figure 1 Here, the module of the apparent age is decomposed into two vectors: its real part and its imaginary part.

3.1. Approximation of the Real Part of the Age of the Universe

Let's perform the approximation of the real part T_{hu} of the age of the universe T_u .

For a radius of curvature r_h of the horizon, the following square root equals zero:

$$z(r_h) = \sqrt{k^4 \cdot r_h^2 - 4 \cdot G^2 \cdot m_u^2} = 0 \quad (16)$$

So, according to the equations (12) and (13), we obtain:

$$T_{hu} = \frac{z(r_u) + 2 \cdot G \cdot m_u \cdot \ln\left(2 \cdot \left[k^2 \cdot r_u + z(r_u)\right]\right)}{\beta \cdot k^3} - \frac{2 \cdot G \cdot m_u \cdot \ln\left(2 \cdot k^2 \cdot r_h\right)}{\beta \cdot k^3} \quad (17)$$

$$\text{where } z(r_u) = \sqrt{k^4 \cdot r_u^2 - 4 \cdot G^2 \cdot m_u^2}$$

Using the equation (10) and by doing a few simplifications, we obtain:

$$T_{hu} = \frac{1}{\beta \cdot k} \left(\sqrt{r_u^2 - r_h^2} + r_h \cdot \ln \left(\frac{\sqrt{r_u^2 - r_h^2} + r_u}{r_h} \right) \right) \quad (18)$$

This same equation could be rewritten in this manner without changing anything: (19)

$$T_{hu} = \frac{r_u}{2 \cdot \beta \cdot c} \cdot \left(1 + \frac{\beta}{2} \right) \cdot \underbrace{\left[\frac{2 \cdot c}{k \cdot r_u \cdot \left(1 + \frac{\beta}{2} \right)} \cdot \left(\sqrt{r_u^2 - r_h^2} + r_h \cdot \ln \left(\frac{\sqrt{r_u^2 - r_h^2} + r_u}{r_h} \right) \right) \right]}_{\approx 1}$$

As shown, the content of the bracket is approximately equal to 1. By doing this approximation and using the equation (7), the equation (19) becomes:

$$T_{hu} \approx \frac{1}{2 \cdot H_0} \left(1 + \frac{\beta}{2} \right) \quad (20)$$

We will use this equation to eventually perform the calculation of the apparent age of the universe.

Let us show that this equation can be further approximated to obtain an equation commonly used by some astronomers to calculate the actual age of the universe.

According to equation (4), the value of $\beta \approx 0.76$. Let's use this approximation to rewrite the equation (20). After a few simplifications, we get:

$$T_{hu} \approx \frac{1}{H_0} \left(\frac{69}{100} \right) \approx \frac{2}{3} \cdot \frac{1}{H_0} \quad (21)$$

This equation can be deduced from the model of Friedman-Lemaître-Robertson-Walker [4,5,6,7,8,9]. Therefore, the equations (20) and (21) represent good approximations of the real part of the age of the universe.

3.2. Approximation of the Imaginary Part of the Age of the Universe

Now, let's find the approximated value of the imaginary part T_{0h} of the age of the universe T_u . From the equation (12), (13) and (16), we get:

$$T_{0h} = \left[\frac{2 \cdot G \cdot m_u \cdot \ln(2 \cdot k^2 \cdot r_h)}{\beta \cdot k^3} \right] - \left[\frac{\sqrt{-4 \cdot G^2 \cdot m_u^2} + 2 \cdot G \cdot m_u \cdot \ln(2 \cdot \sqrt{-4 \cdot G^2 \cdot m_u^2})}{\beta \cdot k^3} \right] \quad (22)$$

Using the equation (10) and performing a few simplifications, we get:

$$T_{0h} = \frac{r_h}{\beta \cdot k} \cdot [\ln(-i) - i] \quad \text{where } i = \sqrt{-1} \quad (23)$$

Using the following relation:

$$\ln(-i) = -\frac{\pi}{2} \cdot i \quad (24)$$

The equation (23), which is the result of a purely imaginary type, can be rewritten:

$$T_{0h} = -\frac{r_h}{\beta \cdot k} \cdot \left[1 + \frac{\pi}{2} \right] \cdot i \quad (25)$$

We can rewrite the equation (25) in the following manner without changing anything:

$$T_{0h} = -\frac{r_u}{2 \cdot \beta \cdot c} \cdot \left(1 - \frac{\beta}{2} \right) \cdot \left(1 + \frac{\pi}{2} \right) \cdot \underbrace{\left[\frac{2 \cdot c}{k} \cdot \frac{r_h}{ru \cdot \left(1 - \frac{\beta}{2} \right)} \right]}_{\approx 1} \cdot i \quad (26)$$

As shown, the content of the parentheses is approximately equal to 1. The equation (19) then becomes:

$$T_{0h} = -\frac{1}{2 \cdot H_0} \cdot \left(1 - \frac{\beta}{2} \right) \cdot \left(1 + \frac{\pi}{2} \right) \cdot i \quad (27)$$

3.3. Approximation of the Apparent Age of the Universe

Let's calculate the apparent age of the universe by using the equation (15) with the equations (20) and (27). After a few simplifications and a development of $1/H_0$, we obtain:

$$|T_u| = |T_{hu} + T_{0h}| = \sqrt{T_{hu}^2 + T_{0h}^2} \approx \frac{1}{H_0} \cdot \underbrace{\left(\frac{1}{2} \cdot \sqrt{\left(1 + \frac{\beta}{2}\right)^2 + \left(\left(1 - \frac{\beta}{2}\right) \cdot \left(1 + \frac{\pi}{2}\right)\right)^2} \right)}_{\approx 1} \quad (28)$$

The content of the parenthesis of the equation (28) is approximately equal to 1. According to equation (7), the equation (28) then becomes approximately equal to equation (2):

$$|T_u| \approx \frac{1}{H_0} \quad (29)$$

Consequently, we have shown that the integral of the equation (12) can be approximated by the equation (2). According to us, based on the approximation calculation made at the equation (29), the equation (2) represents only an apparent age of the universe. In fact, it comes from the calculation of the module of a complex sum of the real part and the imaginary part of the age of the universe.

4. CONCLUSION

In this work, we limited ourselves to calculate the age of the universe. We did not make any conjecture in regard to what happens within the limits of the horizon of the universe.

We have shown that the value $1/H_0$ is a valid approximation of the apparent age of the universe. Assuming that this value corresponds to the real part of the age of the universe, physicists assume incorrectly, we believe, that the speed of light in vacuum was constant over time.

We have shown that the value of $2/(3 \cdot H_0)$ calculated by some physicists corresponds to an approximation of the actual age of the universe. It also represents the best estimate of the true age of the universe.

The originality of our work is that we have calculated the age of the universe using the integral of the inverse of the speed of light as a function of the apparent radius of curvature of the universe. The only required parameters to perform this calculation are c , G and H_0 . This calculation would not have been possible

without the hypothesis formulated in the work cited in [11]. We showed in the latter, among others, that the speed of light has never been constant over time.

The second original point of our work was to conciliate the fact that certain physicists calculated the age of the universe by using the equation $1/H_0$ and that others used the equation $2/(3 \cdot H_0)$. This conciliation is allowed only if we become aware that the age of the universe is in fact of a complex type. The module of the apparent age can be well and truly approximated by $1/H_0$ while the real part of the age of the universe can be approximated by $2/(3 \cdot H_0)$.

Our work raises questions that will require deeper study:

- Why is the portion of time between the big bang and the radius of the horizon of an imaginary type?
- Is the universe really born from a singular point?
- What is happening within the limits of the horizon of the universe?
- How does the universe manage to make the breakthrough on the horizon since the speed of light there was zero?

Despite the radical change in concept that we made, we find that our calculations are fully consistent with the values of the age of the universe commonly used by astrophysicists. This fact reinforces the possibility that our assumptions made in our previous work are correct [11].

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6. APPENDIX A (Added October 13th 2014, revised May 2nd 2019)

Without changing the entire document that precedes the appendix, we want to bring a few precisions on the Hubble constant H_0 . This "constant", which isn't one, varies slowly over time. Since its discovery by Hubble, many ways have been used to measure it using the different observation made by telescope. In spite of all efforts, this parameter, which is essential to describe our universe, suffers a huge lack of precision. Effectively, different measurements made by different teams show that this parameter is probably between 69 and 77 km/(s·MParsec). All observers propose incertitude margins which do not match. However, without having a measurement that joins everybody together and as long as something better is being proposed, a certain consensus is made around a value of 72 km/(s·MParsec). Most serious astrophysics books use this value.

Since new calculations on the universal gravitational constant G we have been able to measure precisely the Hubble constant [19]:

$$H_0 = \frac{c \cdot \alpha^{19} \cdot \beta^{1/2}}{r_e} \approx 72.09548580(32) \text{ km/(s} \cdot \text{MParsec)} \quad (30)$$

The following values come from the CODATA 2010 [20]:

- The speed of light in vacuum $c \approx 299792458 \text{ m/s}$
- The fine structure constant $\alpha \approx 7.2973525698 \times 10^{-3}$
- The classical radius of the electron $r_e \approx 2.8179403267 \times 10^{-15} \text{ m}$

The constant β represents the ratio between the expansion speed of the material universe and the speed of light (see equation (4)).

The value shown in equation (30) is confirmed by measurements made by the Salvatelli [21] who obtained $H_0 \approx 72.1^{+3.2}_{-2.3} \text{ km} \cdot \text{s}^{-1} \cdot \text{MParsec}^{-1}$.

The apparent age of the universe is given by:

$$\left| T_u \right| = \frac{1}{H_0} \quad (31)$$

The MParsec is a unit of distance defined as following:

$$1 \text{ MParsec} \approx 3.085677581 \times 10^{22} \text{ m} \quad (32)$$

The apparent age of the universe, in years, is given by the following equation:

(33)

$$|T_u| \approx \frac{s \cdot \text{MPasec}}{72.09548580 \text{ km}} \cdot \frac{3.085677581 \times 10^{22} \text{ m}}{1 \text{ MParsec}} \cdot \frac{1 \text{ km}}{1000 \text{ m}} \cdot \frac{1 \text{ year}}{365.25 \text{ days}} \cdot \frac{1 \text{ day}}{24 \text{ h}} \cdot \frac{1 \text{ h}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ s}}$$

The value of the **apparent** age of the universe is therefore:

$$|T_u| \approx 1.356 \times 10^{10} \text{ years} = 13.562 \text{ billion years} \quad (34)$$

Let's remind that, according to our model of the universe, the true age of the universe is of complex type.