Calculation of the Quantum of Angle

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Just as there is a quantum of length (Planck length) and a quantum of time (Planck time Planck), we claim that there is a quantum of angle which is related to the deep nature of the fine-structure constant α of a rotating universe with a tangential speed equal to that of light.

In this article, we find the quantum of angle $\varepsilon_\theta \approx 1{,}59\times10^{-122}$ radians linking with the quantum of speed $\varepsilon_v \approx 2{,}38\times10^{-114}$ m·s⁻¹ discovered in the past [1, 2].

KEYWORDS: Quantum of angle, the quantum of speed

1. INTRODUCTION

In mathematics, we meet real and integer numbers. However, in our physical universe, everything seems quantized. So, it makes sense to think that angles are also multiples of a quantum of angle ε_θ.

In our process to calculate the value of the quantum of angle, we will start by finding the smallest existing angle in a static universe that has the apparent dimensions of our universe [2, 3]. Thereafter, wishing to consider the real case of our rotating universe [4], we will use certain results of special relativity concerning Einstein's rotating disks [5] as well as the quantum of speed ε_v (which we already found in the past [1, 3]) to determine the angle quantum ε_θ.

2. Values of the Physics Parameters Used

We will use the compact form of notation to display the tolerances (for example, 2.736 (17) °K will mean 2.736 ± 0,017 °K). The following parameters come from CODATA 2014 [6].

Speed of light in vacuum	$c \approx 299792458$ m·s^{-1}
Planck constant	$h \approx 6.626070040(81)\times10^{-34}$ J·s
Planck length	$L_p \approx 1.616229(38)\times10^{-35}$ m
Planck time	$t_p \approx 5.39116(13)\times10^{-44}$ s
Planck mass	$m_p \approx 2.176\,470(51)\times10^{-8}$ kg
Universal gravitational constant	$G \approx 6.67408(31)\times10^{-11}$ m^3·kg^{-1}·s^{-2}
Fine-structure constant	$\alpha \approx 7.2973525664(17)$

3. The Smallest Angle that May Exist in a Static Universe

If the universe were static (however this is not the case [4]), the smallest angle we would be able to reproduce would be the one that would describe an arc of length equal to Planck length L_p [7] from an apparent radius R_u described from the center of mass of the universe. R_u represents the apparent radius of the universe [2, 3], often called "Hubble radius" [7]. Its value is given by the following equation:

$$R_u = \frac{c}{H_0} \approx 1.28\times10^{26} \text{ m} \tag{1}$$

The value of H_0 represents Hubble constant [8] which is about 72.1 km/(s·MParsec) [3]. This value is also compatible with that of Salvatelli who obtains $H_0 \approx 72.1^{+3.2}_{-2.3}$ km·s^{-1}·MParsec^{-1} [9].

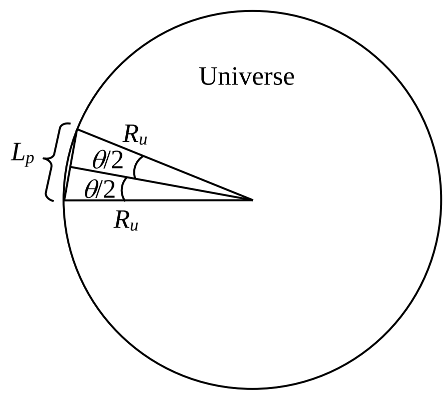

Figure 1) The arc of a circle merges with the segment L_p

For an extremely small angle θ, the line segment of Planck length L_p merges with the arc of a circle and can be determined by the following equation.

$$2R_u \sin\frac{\theta}{2} = L_p \tag{2}$$

Let's recall that Planck length L_p is a quantum of length which comes from Heisenberg's uncertainty principle [10]. It is the smallest possible unit of length. It is therefore normal for this line segment to merge with the arc of a circle described by the universe. In fact, at the macroscopic level, the universe seems circular in appearance. However, a slice of the universe "sphere" is a polygon with so many sides that it looks like a circle.

Planck length L_p is normally defined as a function of the Planck constant h, the speed of light c and the universal gravitational constant G as follows [7]:

$$L_p = \sqrt{\frac{hG}{2\pi c^3}} \approx 1.62\times10^{-35}\,\mathrm{m} \tag{3}$$

For a very small angle, the sine of this angle is the angle itself, in radians. Equation (2) then becomes equation (4).

$$R_u\theta = L_p \rightarrow \theta = \frac{L_p}{R_u} = \frac{1}{\sqrt{N}} = \alpha^{\frac{57}{2}} \approx 1.26\times10^{-61} \text{ radians} \tag{4}$$

Let’s note that due to the dimensions involved, we would have obtained the same result from a single right triangle with an angle θ instead of two triangles with angles $\theta/2$ as described in Figure 1).

In equation (4), N represents the maximum number of lowest energy photons (of wavelength equal to the apparent circumference of the universe $\lambda = 2\pi R_u$) [3]. This number also corresponds to one of the large numbers of Dirac [3, 11]. In the past, we have found more than a hundred equations giving precisely this large number N [12].

$$N = \frac{1}{\alpha^{57}} \approx 6.30\times10^{121} \tag{5}$$

Let’s recall here that we assumed that the universe was static. As mentioned, this is not the case [4]. It rotates with a tangential speed equal to that of light.

4. Einstein’s Disk

Einstein showed that a disc that rotates at a relativistic speed sees its circumference increasing because each line segment which constitutes it contracts by relativistic effect [5]. It is then possible to increase the number of line segments to describe the rotation circle. On the other hand, the angle θ is also seen contracted in a relativistic way since the line segment contracts.

For an observer at rest, a line segment of length L_0 (at rest), will seem to contract by relativistic effect at a length L' when it is accelerated at a speed v [5, 13]. Here, the speed of light in vacuum c plays the role of an unreachable limit speed.

$$L' = L_0 \sqrt{1 - \frac{v^2}{c^2}} \tag{6}$$

According to this equation, if v tends to c, L' will tend to zero. At first glance, this seems to make some sense. However, in special relativity, the corollary equation for mass [5, 13] no longer makes sense at this speed. Indeed, for an observer at rest, a mass at rest m_0 which is accelerated at a speed v will seem to become m' by the same relativistic effect.

$$m' = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{7}$$

Unlike equation (6) where the length L_0 decreases by increasing the speed v, equation (7) shows that the mass m_0 increases when the speed v increases. The problem is that if v tends to c, m' tends to infinity, which is impossible. Indeed, it is absurd to claim to be able to give more kinetic energy to any mass than the energy contained in the total apparent mass m_u of the universe [1, 2]. What is more, we will show that it is impossible to give more energy to an elementary particle than the energy contained in the Planck mass m_p [1, 2]. This second observation is the most restrictive. This places a limit on the speed v.

5. The Quantum of Speed ε_v

In the past, we have shown that there exists a quantum of speed ε_v which represents the smallest unit of speed that may exist [1, 2].

$$\varepsilon_v = \frac{c}{2N} \approx 2.34 \times 10^{-114} \text{ m/s} \tag{8}$$

We have shown that even photons do not move at the limit speed of light c, but slightly below, at c-ε_v [1, 2]. This seems to be the same speed given the smallness of ε_v. However, if we accelerate a Planck mass m_p at the speed $v = c-\varepsilon_v$ with respect to an observer at rest, we would obtain, thanks to equation (7), exactly the apparent mass $m_u \approx 1.73\times10^{53}$ kg of the universe [2, 3, 14]. It makes a lot more sense than an infinite mass.

The apparent mass m_u of the universe is given as a function of the speed of light c, the gravitational constant G and Hubble constant H_0 by the following equation [2, 3, 14]:

$$m_u = \frac{c^3}{GH_0} \approx 1.74 \times 10^{53} \text{ kg} \tag{9}$$

Similarly, for an observer at rest, a particle having a mass at rest $m_0 = m_{ph}$ accelerated at the speed $v = c\text{-}\varepsilon_v$ would have an apparent mass equal to Planck mass m_p.

The mass m_{ph} is the mass of the least energetic photon m_{ph} (which has a wavelength λ equal to that of the apparent circumference of the universe $2\pi R_u$) [2, 3]. The value of R_u is the apparent radius of the universe [2, 3].

$$m_{ph} = \frac{h}{2\pi R_u c} \approx 2.72 \times 10^{-69} \text{ kg} \tag{10}$$

Planck mass m_p represents the highest level of energy that may be given to any particle. Indeed, it is the maximum mass that can be reached when the wavelength is the smallest, that is to say when it is equal to the circumference of a Planck particle ($\lambda = 2\pi L_p$). By substituting the value of L_p by equation (3), we obtain the standard definition of Planck mass m_p:

$$m_p = \frac{h}{2\pi L_p c} = \sqrt{\frac{hc}{2\pi G}} \approx 2.18 \times 10^{-9} \text{ kg} \tag{11}$$

6. Our Universe is Rotating with a Tangential Speed Equal to the Speed of Light

Now, let’s suppose the real case of our universe. According to Stephen Hawking, our universe is in rotation [4].

To convince ourselves that this is the real situation that prevails, imagine a rod of infinite length, infinite rigidity, and zero mass. Let's try to hold this rod motionless. Even with all the goodwill in the world, your hand will shake a little. In your proximity, the rod may seem motionless, but the trembling movement will increase as it moves away from you and somewhere on this rod, far from you, the tangential movement of the rod will be at the speed of light.

Assuming that, as an observer, you start a slow movement of rotation on yourself, the radius of the circle described by your movement will be limited by the fact that the tangential speed cannot exceed the speed of light c. Even if the rod is infinitely rigid, it will seem to bend by relativistic effects. The faster you turn on yourself, the smaller is the radius of rotation. The slower you turn on yourself, the greater is the radius of rotation. The rod of infinite length will begin to wind on itself at the circumference of the circle of rotation described by the radius obtained at the point where the speed tangential to the circle of rotation becomes equal to the speed of light c.

Our universe has the apparent radius it has because it spins at such a speed that it takes about 2π times 13.56 billion years [2] to make a full turn on itself. What's more, its rotational speed decreases over time, allowing the universe to expand, much like a skater stretching out her arms while spinning.

7. The Smallest Angle that May Exist in a Rotating Universe

Considering that the universe is rotating with a relativistic tangential speed, we must take this into account in our calculation of the quantum of angle ε_θ initiated in equation (4) using θ, for a static universe. Let us also take into account that the periphery of the universe has a tangential speed equal to $v = c\text{-}\varepsilon_v$. By combining equations (4), (6) and (8), as well as replacing θ by ε_θ.

$$\varepsilon_\theta = \frac{L_p\sqrt{1-\frac{v^2}{c^2}}}{R_u} = \frac{L_p\sqrt{1-\frac{\left(c-\frac{c}{2N}\right)^2}{c^2}}}{R_u} = \frac{L_p\sqrt{1-\left(1-\frac{1}{N}+\frac{1}{4N^2}\right)}}{R_u} \tag{12}$$

In equation (12), the term N^2 is astronomically large. Its inversed value may, therefore, be neglected. We can then approximate the value of θ as follows:

$$\varepsilon_\theta \approx \frac{L_p}{R_u\sqrt{N}} \approx \frac{L_p\alpha^{\frac{57}{2}}}{R_u} \tag{13}$$

We already know that:

$$\frac{L_p}{R_u} = \alpha^{\frac{57}{2}} \tag{14}$$

Therefore, the value of the quantum of angle is:

$$\varepsilon_\theta \approx \alpha^{57} \approx \frac{1}{N} \approx 1.59\times10^{-122}\,\text{radians} \tag{15}$$

Although it is not the smallest angle that can exist in mathematics, it is the smallest angle that can be found in nature because it follows from the angle described by a right triangle having as hypotenuse the largest dimension that can exist in the universe (the apparent radius R_u of the universe) as well as on one side of the triangle which has the smallest dimension, the Planck length L_p reduced by relativistic effects at the speed $v = c\text{-}\varepsilon_v$.

8. Conclusion

In this article, we have calculated the smallest angle that can exist in our physical world and which we have called the angle quantum ε_θ. In a quantum universe, it seems reasonable to us that even angles can be quantum.

It is interesting to note that, according to equation (15), the quantum of angle $\varepsilon\theta$ is directly equal to the inverse of the large number N which represents the largest possible number of photons in the universe [3].

Although the result of equation (15) is in radians, we know that this result comes from a length ratio as well as from the fine-structure constant raised to a certain power (see equation (13)). It is a unitless number. Temporal variations in length units do not influence this result. We can, therefore, consider that the angle quantum ε_θ is really a constant.

This article and this result teach us something more about the nature of the fine-structure constant α. It could represent the sinus from an angle. This may be a new path to explore to find an equation that can determine the exact value of the fine-structure constant from a purely geometric equation.

9. Conflicts of Interest

The author declares that he has no conflict of interest in connection with the publication of this article.

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