Calculation of the Moment of the Occurrence of Light in the Universe

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The universe was not always transparent and did not always know light and more generally electromagnetic waves. Indeed, in its beginnings, about 361 108 years after the big bang (according to our calculations), the universe was too dense to allow atoms to build up and to have electrons that change orbitals to emit electromagnetic waves. However, after that time, the light occurred and the universe became transparent to electromagnetic waves for the first time.

This delay in the emission of the first photons ensures that the apparent radius of curvature of the luminous universe is slightly smaller than what it would have been if it had begun to emit photons from the big bang. This ensures that the tangential rotation speed of the luminous universe is not quite the velocity of light c in vacuum, but slightly less, causing the expansion of the universe. Indeed, we can show that an emission of photons from the beginning of the big bang would have prevented the universe from expanding due to relativistic affects.

We will also show that the delay in the emission of photons is related to the fine structure constant of the space which can be interpreted as the sine of an angle.

KEY WORDS: Photons, light, transparency of the universe, fine structure constant

1. INTRODUCTION

At the beginning of its creation, the universe was so dense that we could not talk about matter as we conceive it today, we mean by that matter made of protons, neutrons and electrons. To have an emission of photons, the electrons must be able to change of orbital.

To have emission of photons, the universe had to expend and lose a bit of its density. Many astrophysicists think at around 380 000 years after the big bang, the universe had the required conditions to see the first emissions of photons [13,14].

In this article, we will focus on finding a way to calculate the moment where the universe began to emit the first photons. We will make this calculation based on the fact that light accelerates over time.

2. DEVELOPMENT

2.1. Value of the Physics Parameters Used

Let us start by stating all the basic physics parameters that we intend to use in this article. These values are all available in the CODATA 2014 [1].

 $\begin{array}{ll} \bullet \text{ Universal gravitational constant} & G \approx 6.67408(31) \times 10^{-11} \text{ m}^3/(\text{kg} \cdot \text{s}^2) \\ \bullet \text{ Fine structure constant} & \alpha \approx 7.2973525664(17) \times 10^{-3} \\ \bullet \text{ Classical radius of the electron} & r_e \approx 2.8179403227(19) \times 10^{-15} \text{ m} \\ \bullet \text{ Planck time} & t_p \approx 5.39116(13) \times 10^{-44} \text{ s} \\ \bullet \text{ Speed of light in vacuum} & c \approx 299792458 \text{ m/s} \\ \end{array}$

2.2. Calculation of the Acceleration of Light

We want to give here a brief summary of research that we have done in the past on the acceleration of light over time [8].

Einstein has already shown that a huge mass increases the refractive index of the vacuum around it. Based on the general relativity equations, Schwarzschild [10,11] has been able to quantify the speed of light v_L as a function of a distance r from the center of mass m using the following equation:

$$v_L(r) = \frac{c}{\sqrt{1 + \frac{2G \cdot m}{c^2 \cdot r}}}$$

$$\sqrt{1 - \frac{2G \cdot m}{c^2 \cdot r}}$$

This equation uses the gravitational constant G and the velocity of light in vacuum c.

In 1929, Hubble showed that the universe is expanding. Following his research, he brought out a parameter of the universe which he called the "Hubble constant." We should rather speak of a pseudo-constant since its inverse represents the apparent age of the universe. This parameter is therefore called to evolve over time. However, for a short period of time compared to the age of the universe, this parameter may appear to be constant.

The value of the Hubble constant is difficult to measure and in spite of many efforts, astrophysicists still find values between 67 to 76 km/(s·MParsec). However, the most realistic value was found by the Xiaofeng Wang team [4] with a value of $H_0 \approx 72.1(9)$ km/(s·MParsec).

The value of the Hubble constant H_0 corresponds to the inverse of the apparent age of the universe. It also corresponds to the rotation frequency ω of the universe on itself.

In 1929, Hubble discovered that the universe was expanding [3]. The Hubble constant makes it possible to calculate the apparent radius of curvature of the universe R_u (may bear several different names [5,6,7]):

$$R_u = \frac{c}{H_0} \tag{2}$$

This radius of curvature corresponds to the radius of curvature that an expanding sphere would have if it were to take place at the velocity c from the point of origin. However, this does not mean that the expansion of the universe has been at a constant speed. Indeed, using the current velocity of light and the "apparent" age of the universe (which is the inverse of the Hubble constant H_0), it is as if we were making a kind of mean on expanding speeds over time to get the total path traveled.

Still using the Hubble constant H_0 as well as the universal gravitational constant G and the velocity of light in vacuum c, Carvalho [2] shows that it is possible to evaluate the apparent mass of the universe using the following equation:

$$m_u = \frac{c^3}{G \cdot H_0} \tag{3}$$

Thanks to his 1905 special relativity equations, Einstein was able to show us that it is impossible for any object to reach the speed of light [12].

Knowing that the universe is expanding, it is realistic to think that all the matter of the universe moves away from a center of mass. There may be local movements of rapprochement between objects such as galaxies. However, globally, the galaxies move away from one another over time.

Taken as a whole, the universe is expanding at the speed of light. Although the light of the universe (which we have called the luminous universe) can spread at the speed of light, the expansion movement of the material universe can not be achieved at the same speed. This speed must necessarily be less than the speed of

light, let us say at a speed of $\beta \cdot c$. The β factor represents the ratio between the speed of expansion of the material universe and the speed of light c.

If we seek to know where we are with respect to the growing periphery of the luminous universe, we can calculate the apparent radius of curvature r_u versus the center of mass of the universe by making:

$$r_{u} = \beta \cdot R_{u} = \frac{\beta \cdot c}{H_{0}} \tag{4}$$

This movement of expansion over time makes us realize that by moving away from a center of mass, the refractive index of the vacuum of the expanding universe must necessarily diminish over time. This decrease in the overall refractive index of the universe would allow light to accelerate over time

The speed of the current light would then be a snapshot in time of a slow progression. We can then make the hypothesis that this progression will take the same form as equation (1), but with a limiting speed other than c. Let us take arbitrarily the constant k which will represent this new limit. By taking an expansion of infinite dimension, the speed of light will thus reach, after an infinite delay, the value of k.

Let us rewrite equation (1) using the value of k:

$$v_{L}(r) = \frac{k}{\sqrt{1 + \frac{2G \cdot m}{k^{2} \cdot r}}}$$

$$\sqrt{1 - \frac{2G \cdot m}{k^{2} \cdot r}}$$
(5)

In this equation, for a value $r = r_u$ (that is to say, which corresponds to our location in the universe with respect to the center of mass of the universe) and for a mass equal to the apparent mass of the universe m_u , we necessarily get the current speed of light in vacuum c.

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$$v_{L}(r = r_{u}) = c = \frac{k}{1 + \frac{2G \cdot m_{u}}{k^{2} \cdot r_{u}}}$$

$$\sqrt{1 - \frac{2G \cdot m_{u}}{k^{2} \cdot r_{u}}}$$

$$\sqrt{1 - \frac{2G \cdot m_{u}}{k^{2} \cdot r_{u}}}$$
(6)

The speed c represents the derivative of the distance with respect to time. As we mentioned above, the material objects of the universe can not move at the speed of light and will always have a delay with respect to light, hence the factor β that we previously introduced. The objects of the universe therefore move at the speed v_m .

To obtain a more general equation, while being in the universe of mass m_u , let us replace the distance r_u by an arbitrary distance r.

$$v_{m}(r) = \frac{\beta \cdot k}{\sqrt{\frac{1 + \frac{2G \cdot m}{u}}{\frac{k^{2} \cdot r}{2G \cdot m}}} \sqrt{1 - \frac{\frac{2G \cdot m}{u}}{k^{2} \cdot r}}}$$
(7)

By deriving the velocity v_m from equation (7) with respect to the distance r and evaluating it at distance r_u , we obtain exactly the Hubble constant H_0 . Indeed, the Hubble constant is the derivative of the rate of expansion of the material universe (since Hubble used the objects seen by a telescope, here on Earth) with respect to the distance r.

$$H_0 = \frac{dv_m(r)}{dr} \bigg|_{r=r_u} = \frac{\beta \cdot y \cdot k}{r_u} \cdot \left(\frac{1}{(1+y) \cdot \sqrt{1-y^2}}\right)$$
where $y = \frac{2 \cdot G \cdot m_u}{k^2 \cdot r_u}$

So far, the known parameters of the universe we have used are only the velocity of light c, the universal gravitational constant G and the Hubble constant H_0 . We are looking for five unknown parameters: the apparent radius of the universe R_u , the apparent radius of the universe r_u at our location, the apparent mass of the universe m_u , the ultimate velocity of light k when the universe will have an

infinite apparent radius and β which represents the ratio between the expansion velocity of the material universe and the expansion velocity of the luminous universe (which is presently equal to the velocity of light in the vacuum c). By having five unknowns, we need five equations to solve them. Let us use equations (2), (3), (4), (6) and (8) to obtain the following results:

$$R_{u} \approx 1.28 \times 10^{26} \,\mathrm{m} \tag{9}$$

$$r_{u} \approx 9.80 \times 10^{25} \,\mathrm{m} \tag{10}$$

$$m_{\nu} \approx 1.73 \times 10^{53} \text{kg}$$
 (11)

$$k = c\sqrt{2 + \sqrt{5}} \approx 6.17 \times 10^8 \,\text{m/s}$$
 (12)

$$\beta = 3 - \sqrt{5} \approx 0.76 \tag{13}$$

Let us focus on the acceleration of light a_L over time evaluated at the point $r = r_u$. The acceleration of light a_L at our location is the derivative of the velocity of light (from equation (5)) with respect to the time estimated at $r = r_u$. But equation (5) does not contain the time variable, but rather the distance r. Consequently, in order to obtain the acceleration a_L , the derivative of the velocity with respect to the distance r must be used and multiplied by the derivative of the distance r with respect to the time estimated here at $r = r_u$.

$$a_{L}\Big|_{r=r} = \frac{dv_{L}}{dt}\Big|_{r=r} = \left(\frac{dr}{dt} \cdot \frac{dv_{L}}{dr}\right)\Big|_{r=r}$$

$$(14)$$

Since the derivative of the distance r with respect to the time estimated at $r = r_u$ is equal to the speed of light in the current vacuum c, let us substitute in equation (9):

$$a_L\Big|_{r=r} = \left(c \cdot \frac{dv_L}{dr}\right)\Big|_{r=r}$$
 (15)

As a matter of fact, we can evaluate the derivative of the velocity v_L with respect to distance using equation (5) and obtain an equation that would allow us to evaluate a_L at our location

$$a_L \Big|_{r = r_u} = \frac{w \cdot c \cdot k^3}{\sqrt{(x - w) \cdot (x + w)^3}} \approx 9.17 \times 10^{-10} \text{m/s}^2$$
where $x = r_u \cdot k^2$

and
$$w = 2G \cdot m_{u}$$

There is, however, another simpler method for evaluating a_L . Indeed, the derivative of the velocity of light v_L is equal to the derivative of the velocity of expansion of the material universe divided by the factor β . Then, according to equation (8), we obtain:

$$\frac{dv_L}{dr}\bigg|_{r=r_u} = \frac{1}{\beta} \cdot \frac{dv_m}{dr}\bigg|_{r=r_u} = \frac{H_0}{\beta}$$
(17)

Using equation (17), equation (14) allows us to calculate the acceleration of light a_L that can be measured here on Earth at an apparent radius of curvature of the material universe $r = r_u$:

$$a_L \Big|_{r=r_u} = \left(c \cdot \frac{dv_L}{dr} \right) \Big|_{r=r_u} = \frac{c \cdot H_0}{\beta} \approx 9.17 \times 10^{-10} \,\text{m/s}^2$$
 (18)

However, at the periphery of the apparent radius of curvature R_u of the luminous universe, the acceleration of light will be β times smaller:

$$a_L\Big|_{r=R_u} = \left(c \cdot \frac{dv_L}{dr}\right)\Big|_{r=R_u} = c \cdot H_0 \approx 7.00 \times 10^{-10} \,\text{m/s}^2$$
 (19)

2.3. Calculation of the Moment of the Occurrence of Photons

We will calculate here the moment when the first photons appeared in the universe.

If we suppose that the universe has begun to emit photons from the big bang (which is not the case), the luminous universe would have an apparent radius of curvature equal to R_u . However, we know that this could not be the case. Indeed, at the time of the big bang, the density of the universe was too high to allow the formation of atoms. For the first photons to be emitted, it was necessary to wait for the universe to undergo a minimum of expansion, cool a little and leave enough free space between the particles to allow the formation of atoms with their nucleus composed of Neutrons and protons with electrons on orbitals. Only at that time, electrons could emit the first photons by changing orbitals.

This delay in the formation of photons means that the luminous universe has, in reality, an apparent radius slightly less than R_u , which also means that the luminous universe can not be in rotation with a tangential velocity equal to that of the light c, but rather with a slightly lower tangential velocity. However, the radial expansion (parallel to the radius of curvature) of the luminous universe takes place at a speed equal to that of light c.

First, let us show how to make a relativistic summation of two velocity vectors $u_{x,y,z}$ and $w_{x,y,z}$. To simplify the problem, we choose a frame of reference to express the velocity vector $w_{x,y,z}$ so that $w_y = 0$ and $w_z = 0$. To do this, we have to make the rotations and the translations which are required to superimpose the vector $w_{x,y,z}$ with the abscissa axis. It is always possible to do so.

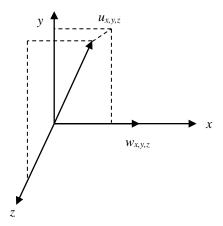


Figure A

The following three equations result in the following vector $v_{x,y,z}$.

$$v_X = \frac{u_X + w_X}{1 + \frac{u_X \cdot w_X}{c^2}} \tag{20}$$

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$$v_{y} = \frac{u_{y}\sqrt{1 - \left(\frac{w_{x}}{c}\right)^{2}}}{1 + \frac{u_{x} \cdot w_{x}}{c^{2}}}$$
(21)

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$$v_{z} = \frac{u_{z}\sqrt{1 - \left(\frac{w_{x}}{c}\right)^{2}}}{1 + \frac{u_{x} \cdot w_{x}}{c^{2}}}$$
(22)

Let us suppose now that locally we simultaneously analyze the expansion and rotation of the universe. Let us consider the case where the luminous universe is expanding at the velocity of light c on the ordinate axis, that is to say $u_x = 0$, $u_y = c$ and $u_z = 0$. Let us suppose also that, on an arbitrary manner, the universe is also rotating on the abscissa axis with a tangential velocity w_x and that $u_y = 0$ and $u_z = 0$ (as explained above).

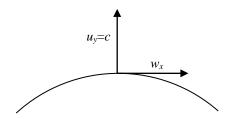


Figure B

Apply this new data to equations (20), (21) and (22). The equations simplify to obtain:

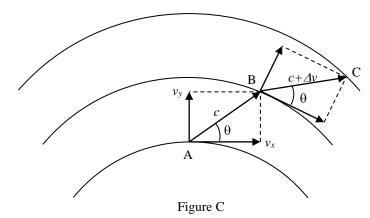
$$v_{r} = w_{r} \tag{23}$$

$$v_x = w_x$$
 (23)
 $v_y = \sqrt{c^2 - w_x^2}$ (24)

$$v_{\mathcal{Z}} = 0 \tag{25}$$

$$v_z = 0$$
If we calculate the module $|v_{x,y,z}|$ of the resulting vector, we obtain:
$$|v_{x,y,z}| = \sqrt{v_x^2 + v_y^2 + v_z^2} = c$$
(25)

This means that a relativistic summation of two velocity vectors, one of which is arbitrary and the other is the velocity of light, gives as a resulting velocity vector which moves at the speed of light c with a direction influenced by the starting velocity vector.



At present, the expansion speed of the sphere of the universe is at the speed of light c. But since the sphere is also rotating at the speed v_x , the point A will be to the point B after a time equal to the Planck time t_p . At each additional Planck time variation, the light will accelerate a little, with a value Δv , and the same process will continue to arrive at point C, and so on.

With the help of equation (19) (because we are at the periphery of the luminous universe), let us evaluate the value of Δv :

$$\Delta v = a_L \cdot t_p = c \cdot H_0 \cdot t_p \approx 3.78 \times 10^{-53} \text{m/s}$$
 (27)

It is a relatively small value, however, it is also for a Planck time $t_p \approx 5.39 \times 10^{-44}$ s that is very small, so that this value still influences the speed of light over time. In the end, the speed of light, at the periphery of the luminous universe, is currently increasing by about 1 m/s every 45.24 years. However, at our location in the universe, the speed of light is currently increasing by about 1 m/s every 34.56 years. Of course, there is a β factor between these two values due to equations (18) and (19).

If we follow the progression in time and space of a given point located at the periphery of the luminous universe, we shall find that it will move by performing a growing spin. This spin will grow due to the angle θ .

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Let us look to the sine of the angle θ . In Figure C, we find that:

$$c \cdot \sin \theta = v_y = \sqrt{c^2 - v_x^2} = c\sqrt{1 - \frac{v_x^2}{c^2}}$$
 (28)

Therefore:

$$\sin \theta = \sqrt{1 - \frac{v_X^2}{c^2}} \tag{29}$$

In the past we have compared the universe to an electron that rotates on itself and have assumed that the fine structure constant α was exactly equal to the Lorentz factor for a tangential rotational velocity v_x :

$$\alpha = \sqrt{1 - \frac{v_x^2}{c^2}} \tag{30}$$

Using equations (29) and (30), we find that:

$$\sin\theta = \alpha \tag{31}$$

By isolating the velocity v_x of equation (30), we obtain:

$$v_x = c\sqrt{1 - \alpha^2} \approx 0.999973 \cdot c$$
 (32)

This means that the periphery of the luminous universe rotates with a tangential velocity very close to the speed of light without being exactly the speed of light.

We could also calculate this velocity by using the acceleration of light a_L at the periphery of the luminous universe. Indeed, the speed v_x is equal to the speed of light minus the acceleration it has undergone during a time Δt . We must use equation (19) to determine the a_L acceleration at the periphery of the luminous universe:

$$v_x = c - a_L \cdot \Delta t = c \cdot (1 - H_0 \cdot \Delta t)$$
(33)

Let us equate equations (32) and (33) to obtain:

$$\sqrt{1-\alpha^2} = 1 - H_0 \cdot \Delta t \tag{34}$$

Let us isolate the time value Δt to obtain:

$$\Delta t = \frac{1 - \sqrt{1 - \alpha^2}}{H_0} \tag{35}$$

In this equation, the fine structure constant α is known very precisely. However, the value of the Hubble constant H_0 is currently a parameter of the universe which is unfortunately evaluated with relatively little precision. In previous work [9], however, we have shown that the Hubble constant can be precisely

determined by using the following equation which we re-evaluated using the CODATA 2014 [1]:

$$H_0 = \frac{c \cdot \alpha^{19} \cdot \beta^{1/2}}{r_e} \approx 72.09554815(32) \,\text{km/(s·MParsec)}$$
 (36)

This value is partly verified by the team of Xiaofeng Wang [4] who measured a value of $H_0 \approx 72.1(9)$ km/(s·MParsec).

Using equation (36), let us evaluate equation (35):

$$\Delta t = r_e \cdot \left(\frac{1 - \sqrt{1 - \alpha^2}}{c \cdot \alpha^{19} \cdot \beta^{1/2}} \right) \approx 361108 \text{ years}$$
 (37)

We can approximate equations (35) and (36) like this:

$$\Delta t \approx \frac{\alpha^2}{2H_0} = \frac{r_e}{2c \cdot \alpha^{17} \cdot \beta^{1/2}}$$
 (38)

This equation means that if we go back in the past, at the very beginning of the expansion of the universe, it took 361 108 years (of our present time) for the first photons of light to be emitted.

This value is approximately equal to the 380 000 years proposed by several physics articles [13,14].

We now know that the universe is expanding and that this expansion is the main cause of the acceleration of light over time. The calculation of the apparent radius of the luminous universe gives us the value R_u . However, this value gives us the apparent radius of the universe if the emission of photons had begun at the beginning of the big bang. For a radius R_u , the tangential velocity would be exactly the velocity of light c. But since there is a delay in the emission of photons, the true apparent radius of the universe is slightly smaller than R_u and the rotation speed is less than the speed of light, i.e. about $0.999973 \cdot c$.

Hypothetically, if there had been emission of photons from the big bang, the light universe would have been expanding at the speed of light c on the ordinate axis, i.e. $u_x = 0$, $u_y = c$ and $u_z = 0$ and the universe would also have been rotating at the speed of light c on the abscissa axis with a tangential $u_x = c$, $u_y = 0$ and $u_z = 0$. We would then have found that the resulting velocity vector would have been:

$$v_x = c (39)$$

$$v_{\mathcal{V}} = 0 \tag{40}$$

$$v_7 = 0 \tag{41}$$

Therefore, without a non-zero fine structure constant, we would have a rotating universe that is not expanding, whatever the value of the expansion rate u_y used. Of course, if it has never been in expansion, it is because such a universe would be of the smallest existing dimension, that is to say with an apparent radius equal to the Planck length L_p . This may seem extremely bizarre as a result, but it is a relativist effect.

Since the particles constituting atoms seem to always have the same dimensions, it is because the variations of dimensions over time are very small. In fact, the particles are rotating on themselves with such a large angular velocity that their apparent radius is of very small value. Like the universe, they rotate with a tangential velocity that is close to that of light. However, in all proportions, the angle θ of these particles is much smaller, which explains why the expansion of the elementary particles over time occurs at a much slower rate than that of the universe. Indeed, if the universe is expanding, it is because in the infinitely small, all the constituents of the universe are also expanding at a rate that is proportional to their relative size in relation to the universe. It is a bit like when a sponge that has been crushed slowly relaxes. The total size of the sponge is directly related to the size of the bubbles that constitute it.

3. CONCLUSION

Using our model of the universe [8], we calculated the acceleration of light over time. This same value made it possible to evaluate the moment when the universe began to be transparent and to emit the first photons shortly after the big bang, i.e. after 361 108 years.

This article shows that there is a link between the moment of the first emission of photons (and the transparency of the universe) and the fine structure constant α . Indeed, we can consider that the first emission of photons is late with respect to the beginning of the expansion of the universe. This delay causes the universe to have an apparent radius slightly smaller than R_u , so that its tangential rotation speed is somewhat less than that of light c.

Perhaps by digging further the reasons why the fine structure constant is non-zero and is associated with the first moments of the transparency of the universe, we would be able to accurately determine the fine structure constant α in finding a geometrical ratio. This article will certainly shed new light on the purpose of the fine structure constant and its exact value.

4. REFERENCES

- "CODATA Recommended Values of the Fundamental Physical Constants: 2014", Cornell University Library, July 2015, Internet paper at: http://arxiv.org/pdf/1507.07956v1.pdf
- [2] Carvalho, Joel C., "Derivation of the Mass of the Observable Universe", *International Journal of Theoretical Physics*, v. 34, no 12, December 1995, p. 2507.
- [3] Hubble, E. and Humason, M. L., "The Velocity-Distance Relation among Extra-Galactic Nebulae", *The Astrophysical Journal*, v. 74, 1931, p.43.
- [4] Wang, Xiaofeng and al., "Determination of the Hubble Constant, the Intrinsic Scatter of Luminosities of Type Ia SNe, and Evidence for Non-Standard Dust in Other Galaxies", March 2011, pp. 1-40, arXiv:astro-ph/0603392v3
- [5] Vargas, J. G. and D.G. Torr, "Gravitation and Cosmology: From the Hubble Radius to the Planck Scale", Springer, v. 126, 2003, pp. 10.
- [6] Sepulveda, L. Eric, "Can We Already Estimate the Radius of the Universe", American Astronomical Society, 1993, p. 796, paragraph 5.17.
- [7] Silberstein, Ludwik, "The Size of the Universe: Attempt at a Determination of the Curvature Radius of Spacetime", *Science*, v. 72, November 1930, p. 479-480.
- [8] Mercier, Claude, "The Speed of Light may not be Constant", Pragtec, Baie-Comeau, Quebec, Canada, October 8th, 2011, paper available on Internet at: www.pragtec.com/physique/
- [9] Mercier, Claude, "Calculation of the Universal Gravitational Constant G", Pragtec, Baie-Comeau, Quebec, Canada, March 13th, 2013, paper available on Internet at: www.pragtec.com/physique/
- [10] Binney, James and Michael Merrifield, "Galactic astronomy", Princeton University Press, 1998, p. 733, from equation A2.
- [11] Maneghetti, Massimo, "Introduction to Gravitational Lensing, Lecture scripts", *Institut für Theoretische Astrophysik*, Bologna, Italy, 2006, p. 7, from equation 1.19, Web. http://www.ita.uni-heidelberg.de/~massimo/sub/Lectures/chapter1.pdf
- [12] Einstein, Albert, "On the Electrodynamics of Moving Bodies", *The Principle of Relativity (Dover Books on Physics)*, New York, publications Dover, 1952 (original 1905 paper), pp. 35-65.
- [13] I. F., Mirabel and al., "Stellar Black Hole at the Dawn of the Universe", Astronomy & Astrophysics, April 2011, vol. 528, id. A149, 7 p.
- [14] Scott, Dodelson, Vesterineau Mike, "Cosmic Neutrino Last Scattering Surface", Physical Review Letters, October 2009, vol. 103, no 17, id. 171301, 4 p.