Calculations and Interpretations of the Different Planck Units

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Planck units are part of a unit system called "natural". The names of these units have been given in honor of Max Karl Ernst Planck and in recognition of his services rendered to the advancement of Physics by his discovery of energy quanta.

These units are usually given from the basic physical constants, such as the Planck constant h, the speed of light in vacuum c, the universal gravitational constant G, Boltzmann's constant k_B, and the permittivity of vacuum ε_0.

We will see that besides being a useful unit system, Planck units always represent a physical reality in which a physical parameter is optimized. We will also see that it is possible to calculate more precisely the units using other physical constants that are not commonly used to describe these units.

KEY WORDS : Planck units

1. INTRODUCTION

Planck units may seem to be the result of a numerology exercise where we put together different basic physics constants to obtain values with units that we were looking for. But its is not the case. Planck units all stem from the fact that our universe is quantum. They can be deduced from the Heisenberg uncertainty principle, which says that it is not possible to know precisely and simultaneously the position and the speed of an object. The simple act of measuring the speed of an object disturb its position and vice versa.

Even if on our scale the space-time continuum of the universe seems to be continuous, its different properties are actually made of tiny "steps". This quantification applies to the time, distances, masses, the energy, etc.

Planck units simplify several physics equations removing conversion factors. Here are some examples (to the left, the basic equations and to the right, the equations shown in Planck units) :

The Newton's equation of the universal gravitation becomes:

$$F=\frac{-G\cdot m_1\cdot m_2}{r^2}\quad\rightarrow\quad F=\frac{-m_1\cdot m_2}{r^2} \tag{1}$$

The Einstein's equation of the energy (from the special relativity) becomes:

$$E = m \cdot c^2 \quad \rightarrow \quad E = \mathrm{m} \tag{2}$$

The Coulomb's law equation becomes:

$$F = \frac{-q_1 \cdot q_2}{4 \cdot \pi \cdot \varepsilon_0 \cdot r^2} \quad \rightarrow \quad F = \frac{-q_1 \cdot q_2}{r^2} \tag{3}$$

And so on...

One of the characteristics of the Planck units is that they can all be defined as a function of one or many of the following constants :

- Speed of light in vacuum c
- Universal gravitational constant G
- Permittivity of the vacuum ε_0
- Planck constant $\hbar$
- Boltzmann constant k_b
- Permeability of the vacuum μ_0

The fact that Planck units are physical limits dictated by the Heisenberg uncertainty principle ensures that all Planck units, basic or secondary, actually represent physical limits or characteristics for which certain parameters are optimized.

The basic Planck units are the Planck time t_p , the Planck length l_p, the Planck m_p, mass T_p and the Planck charge q_p. However, many other secondary units may come from these one.

In this document, we will focus to accurately determine the following Planck units while mentioning how they can represent reality:

- The Planck time t_p
- The Planck length l_p
- The Planck mass m_p
- The Planck temperature T_p
- The Planck charge q_p
- The Planck angular frequency ω_p
- The Planck force F_p
- The Planck pressure p_p
- The Planck energy E_p
- The Planck power ρ_p
- The Planck tension V_p
- The Planck impedance Z_p
- The Planck surface s_p
- The Planck volume v_p

Let's note that obtaining new equations proposed in this document is made possible due to the work that we have done before and that allowed us to determine more accurately the universal gravitational constant G.

2. DEVELOPPEMENT

2.1. The Planck Time t_p

The Planck time is a time unit called "natural" because it only depends on known constants as the gravitational constant G, the Planck constant h and the speed of light c. However, the Planck time is not only a unit used to measure time. Unlike conventional units of time (e.g. seconds), the Planck time is not chosen arbitrarily. It has an intrinsic physical meaning.

In this document, we want to show, among others, where this unit comes from and above all, what it actually means. Then we will use these calculations to determine other Planck units. To do this, we will first demonstrate the origin of the Planck time equation.

Heisenberg enunciated for the first time, in 1927, the uncertainty principle, which is now one of the foundations of quantum mechanics. This principle says that it is not possible to accurately determine the speed and position of an object simultaneously.

In a second statement, Heisenberg teaches us that the uncertainty, in the measurement of the energy of a body, is inversely proportional to the duration of the measurement. Another way to formulate this statement is to say that the energy product ΔE by the time Δt must be:

$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2} \qquad \textbf{(4)}$$

According to the CODATA 2010 [1], the value of the Planck constant is given by $h \approx 6.62606957(29) \times 10^{-34}$ Joule·s. The reduced Planck constant is denoted $\hbar = h / 2 \cdot \pi$.

For a mass at rest m_0, the energy is [12,13]:

$$\Delta E = m_0 \cdot c^2 \qquad \textbf{(5)}$$

According to the CODATA 2010 [1], the speed of light in vacuum is given by $c \approx 299792458$ m/s.

Considering only the case where Δt is the smallest possible amount of time, we re-baptize Δt by t_p and we keep only the equality in **(4)**.

Then, isolating t_p , we obtain:

$$t_p = \frac{\hbar}{2 \cdot m_0 \cdot c^2} \tag{6}$$

Now, let's suppose that we take a photon at rest with a mass m_0, which is at an infinite position, and let's suppose that we confine this photon in a sphere of radius r. The potential energy variation ΔE_p that would be used up would be:

$$\Delta E_p = -G \cdot {m_0}^2 \cdot \left(\frac{1}{r_\infty} - \frac{1}{r} \right) \Bigg|_{r_\infty = \infty} = \frac{G \cdot {m_0}^2}{r} \tag{7}$$

Here, G is the universal gravitational constant.

Choosing r so that the potential energy variation ΔE_p corresponds to half ΔE, we obtain the following special case:

$$\frac{m_0 \cdot c^2}{2} = \frac{G \cdot {m_0}^2}{r} \tag{8}$$

If we isolate r, we get exactly what we call the Schwarzschild radius:

$$r = \frac{2 \cdot G \cdot m_0}{c^2} \tag{9}$$

This is a black hole radius. For this radius, the speed of light becomes zero. Isolating m_0, we get:

$$m_0 \cdot = \frac{r \cdot c^2}{2 \cdot G} \tag{10}$$

Using **(10)** in **(6)**, we obtain:

$$t_p = \frac{\hbar \cdot G}{r \cdot c^4} \tag{11}$$

But the value of r may be obtained by the following relation:

$$r = c \cdot t_p \tag{12}$$

According to the equations **(11)** and **(12)**, the Planck time t_p is defined by:

$$t_p = \sqrt{\frac{\hbar \cdot G}{c^5}} = \sqrt{\frac{h \cdot G}{2 \cdot \pi \cdot c^5}} \tag{13}$$

This equation allows to calculate the smallest measurable unit of time.

According to the CODATA 2010 [1], the value of the Planck time is given by $t_p \approx 5.39106(32) \times 10^{-44}$ s.

The biggest uncertainty in the value of the Planck time t_p is the value of the gravitational constant G, which is extremely difficult to measure. To measure this constant, physicists currently use a torsion balance invented by Cavendish.

To build a torsion balance of Cavendish, a small conductive wire is suspended from the ceiling of a distant metal support (to avoid gravitational interactions). In this balance, everything is made of metal or of conductive materials to conduct electrical charges to ground. This ploy meant to avoid the accumulation of electrical charges that could skew the data by creating an electric field. On the wire is suspended a conductive horizontal bar. At the end thereof are mounted two identical metal masses. These masses are as big as possible while avoiding breaking the suspension wire. Two other fixed masses mounted on a metal support are inserted at a given time. The masses that are suspended on the wire then move toward the other masses which are mounted on the frame. The rotational movement of the masses around the rotation axis which is made of the wire hanger stops when the attraction force between the masses is equal to the torsional force of the wire. This torque reacts exactly like a weak spring. Knowing the constant of this spring and knowing the angle of rotation of the masses, we deduce the gravitational force between the masses. Knowing the value of the masses, we deduce the value of the universal gravitational constant in the Newton equation.

Although the equations of Einstein's general relativity are more accurate than Newton's one to calculate and predict the trajectories of moving masses, the equation of Newton's theory on the gravitation of masses is considered infinitely precise for static forces. Even Einstein's equations are calibrated using Newton's equation for static forces. Just as Newton's equation, the equations of Einstein's general relativity are using the universal gravitation constant G.

In a Cavendish torsion balance, to avoid the wire hanger to break because of the mass weight, the masses involved are relatively small. Even the external masses are forced to be limited by the physical limits. Of course, the masses are made of noble and stable metals (to avoid mass variation over time) with a high density. Due to the low masses involved, the forces generated remain extremely weak. They are difficult to measure accurately, even being extremely cautious. External influences can easily interfere with the balance. For example, the internal vibrations of the Earth (earthquakes, traffic and others), ambient lighting (photons can create an involuntary thrust), room temperature (that can change the torsion constant of the wire), the position of Earth, Moon and Sun (which can create additional forces or counter-forces), etc. In short, even with all the minutiae of the world, it is practically impossible to be in control of all the parameters involved.

In recent years, the office of weights and measures (Bureau des poids et mesures) attempts to calibrate all measuring units with reference to the speed of light in vacuum which is easily repeatable and measurable with high precision thanks to lasers. According to the postulates of relativity, the speed of light is invariant in time. According to our work, we know that this is not entirely true and that the light undergoes a slight acceleration over time. However, despite this, if we could guarantee the accuracy of any mass with the same resolution as that of the speed of light, it would represent a major breakthrough in the world of metrology.

Despite all the standardization efforts, like for the units of time and distance, the definition of mass can not (for now) be related to the speed of light in vacuum. For this reason, the definition of the mass has not improved much over years. We still use a standard that is deteriorating year after year by losing mass due to radioactive isotopes that comprise it and because of the comparison and cleaning manipulations that are made from time to time. The standard kilogram is deteriorating and limits the accuracy of measurements. It is not easily comparable and the repeatability of secondary standards is seriously flawed.

In previous work [11], we found an equation that gives the value of the universal gravitational constant G as a function of physics constants with a similar accuracy than the speed of light in vacuum.

$$G = \frac{c^2 \cdot r_e \cdot \alpha^{20}}{m_e \cdot \beta} \approx 6.673230436(30) \times 10^{-11} \mathrm{m}^3/(\mathrm{kg} \cdot \mathrm{s}^2) \tag{14}$$

According to the CODATA 2010 [1]:

- Universal gravitational constant $G \approx 6.67384(80) \times 10^{-11}$ m^3/(kg·s^2)
- Speed of light in vacuum $c \approx 299792458$ m/s
- Classical radius of the electron $r_e \approx 2.8179403267(27) \times 10^{-15}$ m
- Mass of the electron $m_e \approx 9.10938291(40) \times 10^{-31}$ m
- Fine structure constant $\alpha \approx 7.2973525698(24) \times 10^{-3}$

The value of β is an irrational number. It gives the ratio between the expansion speed of the material universe and the speed of light in vacuum c [10]:

$$\beta = 3 - \sqrt{5} \approx 0.76 \tag{15}$$

To obtain a better accuracy, let's try to express the equation **(13)** without using the gravitational constant G and the Planck constant h.

The Compton radius r_c may be calculated from the following equality between the energy contained in the rest mass of the electron and the wave that is associated with it:

$$m_e \cdot c^2 = \frac{h \cdot c}{2\pi \cdot r_c} \tag{16}$$

The Compton radius may be described as a function of the classical radius of the electron r_e and of the fine structure constant α:

$$r_e = \alpha \cdot r_c \tag{17}$$

Using the equations **(14)**, **(15)** and **(16)** in the equation **(13)**, we obtain:

$$t_p = \frac{r_e}{c}\sqrt{\frac{\alpha^{19}}{\beta}} \approx 5.3908142(52) \times 10^{-44}\,\mathrm{s} \tag{18}$$

This equation is 1200 time more precise than the value of the CODATA 2010 [1] value**.**

Since the Planck time t_p comes from the Heisenberg uncertainty principle that uses the least measurable amount of energy, the Planck time then represents the smallest measurable unit of time. Time is slipping away with blows of steps that accumulate. All time variations are necessarily an integer multiple of the Planck time t_p.

It is interesting to note, without trying to demonstrate it here, that the Planck time t_p can be described in terms of the apparent age of the universe and of the fine structure constant α:

$$t_p = T_u \sqrt{\alpha^{57}} = \frac{1}{H_0}\sqrt{\alpha^{57}} \tag{19}$$

We should know that the apparent age of the universe [2] T_u is equal to the inverse of the Hubble constant H_0 [9] .

$$T_u = \frac{1}{H_0} \tag{20}$$

In previous work, we showed that the Hubble constant may be accurately determined using the following equation [11,14]:

$$H_0 = \frac{c \cdot \alpha^{19} \cdot \beta^{1/2}}{r_e} \approx 72.09548632(46)\ \mathrm{km/(s \cdot MParsec)} \tag{21}$$

This value is partly checked by the of Wang Xiaofeng team [15] who measured a value of $H_0 \approx 72.1\ (9)$ km / (s·MParsec).

2.2. The Planck Length l_p

The Planck length l_p is defined by:

$$l_p = c \cdot t_p = \sqrt{\frac{\hbar \cdot G}{c^3}} \tag{22}$$

According to the CODATA 2010 [1], the value of the Planck length is given by $l_p \approx 1.616199(97) \times 10^{-35}$ m.

In equation **(22)**, the speed of light c represents an unsurpassable upper limit according to the postulates of the relativity. The Planck time t_p is the smallest measurable unit of time. Therefore, the Planck length l_p represents the smallest unit of length that a photon can pass while being measurable. A photon that travels in space will move forward by steps of equal to the Planck length distance. All routes through space are actually an integer multiple of the Planck length l_p.

For better accuracy in the evaluation of the Planck length l_p, let's try to express the equation **(22)** without using the gravitational constant G and without the Planck constant h.

Using the equations **(14)**, **(15)** and **(16)** in the equation **(22)**, we obtain:

$$l_p = r_e \cdot \sqrt{\frac{\alpha^{19}}{\beta}} \approx 1.616125436(53) \times 10^{-35}\,\mathrm{m} \tag{23}$$

This equation is 18000 times more accurate than the CODATA 2010 value [1]**.**

It is interesting to note, without trying to demonstrate it here, that the Planck length l_p may be described in terms of the apparent radius of curvature of the universe R_u and of the fine structure constant α:

$$l_p = R_u \sqrt{\alpha^{57}} = \frac{c}{H_0} \sqrt{\alpha^{57}} \tag{24}$$

2.3. The Planck Mass m_p

The Planck mass m_p is the mass of a particle when it reaches its highest energy level. This property stems from the wave-particle duality of matter.

From a corpuscular point of view, the energy of a particle of mass m_p can be given by the equation of Einstein's relativity:

$$E = m_p \cdot c^2 \tag{25}$$

From a wave standpoint, the energy of a wave with a wavelength equal to $2\pi \cdot l_p$ is given by the following equation:

$$E = \frac{h \cdot c}{2\pi \cdot l_p} \tag{26}$$

By making equal the two last equations, we get:

$$m_p \cdot c^2 = \frac{h \cdot c}{2\pi \cdot l_p} \tag{27}$$

If we replace l_p by the equation **(22)** and that we isolate m_p, we obtain the equation that is described in the CODATA 2010 [1] for the Planck mass m_p is defined by:

$$m_p = \sqrt{\frac{h \cdot c}{2\pi \cdot G}} = \sqrt{\frac{\hbar \cdot c}{G}} \approx 2.17651(13) \times 10^{-8}\,\mathrm{k}g \tag{28}$$

Since this equation is obtained from the smallest possible wavelength (see equation **(26)**), we conclude that this equation represents the highest possible energy level for a particle. Therefore, m_p which comes from the equality of this equation with the equation **(25)**, necessarily represent the largest possible mass for a particle.

It is interesting to note that the Planck mass m_p also corresponds to the geometric mean between the largest existing mass (the apparent mass of the universe m_u) and the smallest existing mass (the mass m_{ph} that is associated to the photon that has the largest wavelength and that has the apparent diameter of the universe $2\pi \cdot R_u$).

The apparent mass of the universe m_u is given by [3,4]:

$$m_u = \frac{c^3}{G \cdot H_0} \tag{29}$$

The mass m_{ph} of a photon with a wavelength $2\pi \cdot R_u$ is given by the following equation:

$$m_{ph} = \frac{h}{2\pi \cdot R_u \cdot c} \tag{30}$$

The apparent radius of curvature of the universe R_u (which may bear various name in different documents) is given by the following equation [5,6,7,8] :

$$R_u = \frac{c}{H_0} \tag{31}$$

Using equations **(28)** to **(31)**, it is possible to show that the geometric mean between the apparent mass of the universe m_u and the mass associated with the lightest photon m_{ph} gives exactly the Planck mass m_p:

$$m_p = \sqrt{m_u \cdot m_{ph}} \tag{32}$$

It is possible to define more accurately the Planck mass m_p than the equation **(28)** using equations **(14)**, **(15)** and **(16)** into equation **(22)**:

$$m_p = m_e \cdot \sqrt{\frac{\beta}{\alpha^{21}}} \approx 2.17660867(10) \times 10^{-8} \text{kg} \tag{33}$$

This value is 1300 times more accurate than the CODATA 2010 [1] one.

2.4. The Planck Temperature T_p

The Planck mass m_p corresponds to the highest energy level possible for a particle that we can meet. Therefore, when the energy of such a particle is converted into pure energy, the recorded temperature is then the highest that it is possible to obtain in the universe.

The energy E contained in a particle having the Planck mass m_p is given by the mass-energy conversion equation of Einstein:

$$E = m_p \cdot c^2 \tag{34}$$

Similarly, the energy E contained in a particle having a temperature T_p is given by:

$$E = k_b \cdot T_p \tag{35}$$

By doing the equality and isolating T_p, we obtain the Planck temperature T_p that can be redefined as follows using equation **(28)**:

$$T_p = \frac{m_p \cdot c^2}{k_b} = \sqrt{\frac{\hbar \cdot c^5}{G \cdot k_b^2}} \approx 1.416833(85) \times 10^{32}\ ^\circ\text{K} \tag{36}$$

It is possible to define the Planck temperature more accurately using the equations **(14)**, **(15)** and **(16)** into equation **(36)**. We then get:

$$T_p = \frac{m_e \cdot c^2}{k_b} \cdot \sqrt{\frac{\beta}{\alpha^{21}}} \approx 1.4168978(13) \times 10^{32}\ °\mathrm{K} \tag{37}$$

This value is 65 times more accurate than the CODATA 2010 [1] one.

2.5. The Planck Charge q_p

Let's begin by showing where the Planck charge comes from to show that it corresponds to the highest charge that a particle may have.

Again, the energy E contained in a particle having the Planck mass m_p is given by the mass-energy conversion equation of Einstein:

$$E = m_p \cdot c^2 \tag{38}$$

We may consider that all the kinetic energy contained in the mass of the particle comes from the electrostatic energy E contained in a point-like particle that has a Planck length l_p radius and a charge q_p. This energy is given by the following equation:

$$E = \frac{q_p^2}{4\pi \cdot \varepsilon_0 \cdot l_p} \tag{39}$$

By doing the equality between the equations **(38)** and **(39)** and isolating the Planck charge q_p, we obtain:

$$q_p = \sqrt{4\pi \cdot m_p \cdot l_p \cdot \varepsilon_0 \cdot c^2} \tag{40}$$

The speed of light is defined as a function of the permeability μ_0 of the vacuum and the permittivity ε_0 of the vacuum as follows:

$$c = \sqrt{\frac{1}{\mu_0 \cdot \varepsilon_0}} \tag{41}$$

It is thus possible to rewrite the equation **(40)** using the equation **(41)**:

$$q_p = \sqrt{\frac{4\pi \cdot m_p \cdot l_p}{\mu_0}} \tag{42}$$

Using equations **(22)** and **(28)** into the equation **(42)** we obtain the Planck charge q_p according to Planck constant h:

$$q_p = \sqrt{\frac{2 \cdot h}{\mu_0 \cdot c}} = \sqrt{\frac{4\pi \cdot \hbar}{\mu_0 \cdot c}} \qquad \textbf{(43)}$$

Let's note that due to the fact that the Planck charge is described here using the Planck mass m_p and the Planck length l_p which both correspond to extremes, we conclude that the Planck charge q_p is the maximum charge that can be measured for a particle.

Now, if we use the equation **(41)** in the equation **(43)**, we obtain:

$$q_p = \sqrt{2c \cdot h \cdot \varepsilon_0} \approx 1.8755460(41) \times 10^{-18} \text{ Coulomb} \qquad \textbf{(44)}$$

It is interesting to note, without trying to demonstrate it here that it is also possible to describe the Planck charge based on the apparent mass of the universe m_u and its apparent radius of curvature R_u:

$$q_p = \sqrt{\frac{4\pi \cdot m_u \cdot R_u \cdot \alpha^{57}}{\mu_0}} \qquad \textbf{(45)}$$

The electron charge q_e can be defined as a function of the mass of the electron m_e , the classical radius of the electron r_e and the permeability of the vacuum μ_0:

$$q_e = \sqrt{\frac{4\pi \cdot m_e \cdot r_e}{\mu_0}} \qquad \textbf{(46)}$$

It is possible to define the Planck charge equally precisely in **(44)** by defining from the electron charge q_e , the mass of the electron m_e , classical radius of the electron r_e , of the fine structure constant α and the permeability of vacuum μ_0 using equations **(14)**, **(15)**, **(16)** and **(46)** into equation **(42)**. We then get:

$$q_p = q_e \cdot \sqrt{\frac{1}{\alpha}} = \sqrt{\frac{4\pi \cdot m_e \cdot r_e}{\mu_0 \cdot \alpha}} \approx 1.8755460(41) \times 10^{-18} \text{ Coulomb} \qquad \textbf{(47)}$$

2.6. The Planck Angular Frequency ω_p

The Planck angular frequency ω_p is defined by:

$$\omega_p = \frac{1}{t_p} = \sqrt{\frac{c^5}{\hbar \cdot G}} \approx 1.85492(11) \times 10^{43} \text{ rad/s} \qquad \textbf{(48)}$$

Since the Planck angular frequency ω_p is the inverse of the Planck time t_p which is the smallest possible unit of time, thus ω_p it represents the maximal angular frequency that a particle may have when it reaches its highest energy level, that is to say, when a particle of radius equal to the Planck length l_p spins so fast that the periphery speed of the particle reaches the speed of light c.

It is interesting to note, without trying to demonstrate it here that the Planck angular frequency ω_p can be described in terms of the apparent age of the universe you and of the fine structure constant α.

$$\omega_p = \frac{1}{T_u} \cdot \sqrt{\frac{1}{\alpha^{57}}} = H_0 \sqrt{\frac{1}{\alpha^{57}}} \tag{49}$$

It is possible to define the Planck angular frequency more accurately by defining it from the speed of light in the vacuum c, the classical radius of the electron r_e , from the fine structure constant α and the β factor using the equations **(14)**, **(15)** and **(16)**:

$$\omega_p = \frac{c}{r_e} \cdot \sqrt{\frac{\beta}{\alpha^{19}}} \approx 1.85500736(61) \times 10^{43} \text{ rad/s} \tag{50}$$

2.7. The Planck Force F_p

The Planck force F_p is defined by:

$$F_p = \frac{m_p \cdot l_p}{t_p^2} = \frac{c^4}{G} \approx 1.21034(15) \times 10^{44} \text{ N} \tag{51}$$

Due to the fact that we use the Planck mass m_p and the Planck time t_p to describe this force, the Planck force F_p corresponds to the greatest force that may be applied on an object.

This evaluation of the Planck force is not very accurate due to the fact that it uses the gravitational constant as defined in the CODATA 2010 [1].

It is interesting to note, without trying to demonstrate here that the force of Planck F_p is equal to the force required to accelerate the apparent mass of the universe m_u over a distance equal to the apparent radius of curvature of the universe R_u for a period of time equal to the apparent age of the universe T_u. By using Newton's equation $F = m{\cdot}a$ (where m is the mass in kg of the object and its acceleration in m/s^2), we obtain:

$$F_p = m_u \cdot \frac{R_u}{T_u^2} = m_u \cdot R_u \cdot H_0^2 = m_u \cdot c \cdot H_0 = \frac{m_u \cdot c^2}{R_u} \tag{52}$$

Since the apparent mass of the universe m_u , the apparent radius of the universe R_u and the apparent age of the universe T_u are undoubtedly the greatest values being for these measurement units, we conclude again that the Planck force F_p is the greatest force that can be applied to an object.

Let's recall that the apparent age of the universe T_u is given by the inverse of the Hubble constant H_0:

$$T_u = \frac{1}{H_0} \approx 72.09548632(46) \text{ km/(s} \cdot \text{MParsec)} \approx 4.27998719(27) \times 10^{17} \text{ s} \tag{53}$$

It is possible to define the Planck force F_p more accurately by defining it from the mass of the electron m_e, the classical electron radius r_e, the fine structure constant α and β using equations **(14)**, **(15)** and **(16)** into equation **(51)**:

$$F_p = \frac{m_e \cdot c^2 \cdot \beta}{r_e \cdot \alpha^{20}} \approx 1.210449555(53) \times 10^{44} \text{ N} \tag{54}$$

2.8. The Planck Pressure p_p

The Planck pressure p_p is defined by:

$$p_p = \frac{F_p}{l_p^2} = \frac{c^7}{\hbar \cdot G^2} \approx 4.6344(11) \times 10^{113} \text{ N/m}^2 \tag{55}$$

Since it stems from the greatest force applied on the smallest surface, this pressure is the biggest pressure that can be exerted on a particle.

It is interesting to note, without trying to demonstrate it here that the Planck pressure p_p is also equal to the pressure would exert the mass of the universe if it had accelerated over a distance equal to the apparent radius of curvature of the universe R_u for a period of time equal to the apparent age of the universe T_u and that this equivalent force was applied on a square surface with sides equal to the Planck length l_p.

$$p_p = \frac{m_u \cdot R_u}{T_u^2 \cdot L_p^2} = \frac{m_u \cdot R_u \cdot H_0^2}{L_p^2} = \frac{m_u \cdot c \cdot H_0}{L_p^2} = \frac{m_u \cdot c^2}{R_u \cdot L_p^2} \tag{56}$$

It is possible to define the Planck pressure p_p more accurately by defining it from the mass of the electron m_e, the classical radius of the electron r_e, the fine structure constant α and β using the equations **(14)**, **(15)** and **(16)** into equation **(55)**:

$$p_p = \frac{m_e \cdot c^2 \cdot \beta^2}{r_e^3 \cdot \alpha^{39}} \approx 4.63443253(21) \times 10^{113}\ \text{N/m}^2 \tag{57}$$

2.9. The Planck Energy E_p

The Planck energy E_p is defined by:

$$E_p = F_p \cdot l_p = m_p \cdot c^2 = \sqrt{\frac{\hbar \cdot c^5}{G}} \approx 1.95615(12) \times 10^9\ \text{J} \tag{58}$$

This energy corresponds to the maximum energy level that may have a particle. Indeed, the ultimate mass for a particle is the Planck mass m_p, and the speed of light in vacuum c represents the upper speed limit.

It is interesting to note, without trying to demonstrate it here that the Planck energy E_p is also equal to the energy that would dissipate the mass m_u of the entire universe if it were converted into pure energy by the Einstein's equation of the special relativity multiplied by the ratio between the Planck length l and the apparent radius of curvature of the universe R_u.

$$E_p = m_u \cdot c^2 \cdot \frac{l_p}{R_u} \tag{59}$$

It is possible to define the Planck energy E_p more accurately by defining it from the mass of the electron m_e, the classical radius of the electron r_e, the fine structure constant α and the β factor using the equations **(14)**, **(15)** and **(16)** into equation **(58)**:

$$E_p = m_e \cdot c^2 \cdot \sqrt{\frac{\beta}{\alpha^{21}}} \approx 1.95623831(86) \times 10^9\ \text{J} \tag{60}$$

2.10. The Planck Power P_p

The Planck Power P_p is defined by:

$$P_p = \frac{E_p}{t_p} = \frac{c^5}{G} \approx 3.62850(43) \times 10^{52}\ \mathrm{W} \tag{61}$$

Since the Planck power is described as a function of the Planck energy representing a maximum energy level for a particle and as a function of the Planck time that is the smallest unit of time, the Planck power is the greatest power that can be delivered.

It is interesting to note, without trying to demonstrate it here that the Planck power can be obtained as a function of the apparent mass of the universe m_u and as a function of the apparent age of the universe T_u (thus depending on the inverse of the Hubble constant H_0):

$$P_p = \frac{m_u \cdot c^2}{T_u} = m_u \cdot c^2 \cdot H_0 \tag{62}$$

It is possible to define the Planck power P_p more accurately by defining it from the mass of the electron m_e, the classical radius of the electron r_e, the fine structure constant α and the β factor using the equations **(14)**, **(15)** and **(16)** into the equation **(61)**.

$$P_p = \frac{m_e \cdot c^3 \cdot \beta}{r_e \cdot \alpha^{20}} \approx 3.62883647(16) \times 10^{52}\ \mathrm{W} \tag{63}$$

2.11. The Planck Density ρ_p

The Planck density ρ_p is defined by:

$$\rho_p = \frac{m_p}{l_p^3} = \frac{c^5}{\hbar \cdot G^2} \approx 5.1556(12) \times 10^{96}\ \mathrm{kg/m^3} \tag{64}$$

Since the Planck density ρ_p is defined according to the Planck mass m_p, which is the largest mass that is available for a particle and that this density is obtained for a cube having edges equal to the Planck length l_p, we conclude that the Planck density ρ_p corresponds to the greatest density possible in the universe.

It is interesting to note, without trying to demonstrate it here, that the Planck density can be described as a function of the apparent mass of the universe m_u and as a function of the apparent radius of curvature of the universe R_u:

$$\rho_p = \frac{m_u}{R_u \cdot \alpha^{57}} = \frac{c^2}{G \cdot R_u^2 \cdot \alpha^{57}} \tag{65}$$

It is possible to define the Planck density ρ_p more accurately by defining it from the mass of the electron m_e , classical radius of the electron r_e , the fine structure constant α and the β factor using the equations **(14)**, **(15)** et **(16)** into the equation **(64)**.

$$\rho_p = \frac{m_e \cdot \beta^2}{r_e^3 \cdot \alpha^{39}} \approx 5.15650161(24) \times 10^{96} \text{ kg/m}^3 \tag{66}$$

2.12. The Planck Current I_p

The Planck current I_p is defined by:

$$I_p = \frac{q_p}{t_p} = \sqrt{\frac{4\pi \cdot c^6 \cdot \varepsilon_0}{G}} \approx 3.47899(21) \times 10^{25} \text{ A} \tag{67}$$

It is interesting to note, without trying to demonstrate it here that it is also possible to describe the Planck current I_p as a function of the apparent mass of the universe m_u , its apparent radius of curvature R_u and the apparent age of the universe T_u (which corresponds to the inverse of the Hubble constant H_0):

$$I_p = \frac{1}{T_u} \cdot \sqrt{\frac{4\pi \cdot m_u \cdot R_u \cdot \alpha^{57}}{\mu_0}} = H_0 \cdot \sqrt{\frac{4\pi \cdot m_u \cdot R_u \cdot \alpha^{57}}{\mu_0}} \tag{68}$$

Again, without trying to demonstrate it here, the Planck current I_p may also be described as a function of the charge of the electron q_e , the apparent age of the universe T_u (therefore from the inverse of the Hubble constant H_0) and the fine structure constant α:

$$I_p = \frac{q_e}{T_u \cdot \alpha^{29}} = \frac{q_e \cdot H_0}{\alpha^{29}} \tag{69}$$

It is possible to define the Planck current I_p more accurately than in **(67)** by defining it from the mass of the electron m_e , the classical radius of the electron r_e , the permeability of vacuum μ_0 , the ratio β and the fine structure constant α using the equations **(14)**, **(15)** and **(16)**. We then get:

$$I_p = \frac{c}{\alpha^{10}} \cdot \sqrt{\frac{4\pi \cdot m_e \cdot \beta}{\mu_0 \cdot r_e}} \approx 3.479151553(77) \times 10^{25} \text{ A} \quad \textbf{(70)}$$

It is also possible to define the Planck current I_p as a function of the charge of the electron q_e , the classical radius of the electron r_e and the fine structure constant α. We then get:

$$I_p = \frac{q_e \cdot c \cdot \beta^{1/2}}{r_e \cdot \alpha^{10}} \approx 3.479151553(77) \times 10^{25} \text{ A} \quad \textbf{(71)}$$

2.13. The Planck Tension V_p

The Planck tension V_p is defined (using CODATA 2010 constants) by:

$$V_p = \frac{E_p}{q_p} = \sqrt{\frac{c^4}{4\pi \cdot G \cdot \varepsilon_0}} \approx 1.042976(63) \times 10^{27} \text{ V} \quad \textbf{(72)}$$

It is interesting to note, without trying to demonstrate it here that it is also possible to describe the Planck tension Vp as a function of the apparent mass of the universe m_u , its apparent radius of curvature R_u and the apparent age of the universe T_u (which corresponds to the inverse of the Hubble constant H_0):

$$V_p = \sqrt{\frac{m_u \cdot R_u \cdot H_0^2}{4\pi \cdot \varepsilon_0}} = \sqrt{\frac{m_u \cdot c \cdot H_0}{4\pi \cdot \varepsilon_0}} = \sqrt{\frac{m_u \cdot c^3 \cdot H_0 \cdot \mu_0}{4\pi}} \quad \textbf{(73)}$$

Again, without trying to demonstrate here, the Planck voltage Vp may also be described as a function of the mass of the universe m_u , the charge of the electron q_e and fine structure constant α:

$$V_p = \frac{m_u \cdot c^2 \cdot \alpha^{59}}{q_e} \quad \textbf{(74)}$$

It is possible to describe the Planck voltage V_p more accurately than in **(72)**, by defining it as a function of the mass of the electron m_e , the classical radius of the electron r_e and the fine structure constant α using the equations **(14)**, **(15)** and **(16)**. We then get:

$$V_p = \frac{c}{\alpha^{10}} \cdot \sqrt{\frac{m_e \cdot \beta}{4\pi \cdot \varepsilon_0 \cdot r_e}} \approx 1.043023397(23) \times 10^{27}\ \mathrm{V} \qquad \textbf{(75)}$$

It is also possible to define the Planck voltage V_p as a function of the charge of the electron q_e, classical electron radius r_e , the permittivity of vacuum ε_0 and fine structure constant α using the equations **(14)**, **(15)** and **(16)**. We then get:

$$V_p = \frac{q_e \cdot \beta^{1/2}}{4\pi \cdot \varepsilon_0 \cdot r_e \cdot \alpha^{10}} \approx 1.043023397(23) \times 10^{27}\ \mathrm{V} \qquad \textbf{(76)}$$

2.14. The Planck Impedance Z_p

The Planck impedance Z_p is defined by:

$$Z_p = \frac{V_p}{I_p} = \frac{1}{4\pi \cdot \varepsilon_0 \cdot c} = \frac{Z_0}{4\pi} \approx 29.986\,\Omega \qquad \textbf{(77)}$$

Curiously, this also corresponds to the impedance of transmission cables for which it is possible to drive through a maximum power. But, this is not the impedance for which the losses are the smallest (which are around 77 Ω (hence the 75 Ω standard coaxial cables for television). The 50 Ω impedance for telecommunication cables is a good compromise between the maximum power that can be driven through with a minimum loss. In fact, the arithmetic mean between the Planck impedance and the impedance giving the minimum loss gives approximately 53.5 Ω and the geometric mean gives about 48 Ω. So, the standardized 50 Ω impedance represents a good compromise.

Since the Planck impedance does not depend on the gravitational constant but it only depends on ε_0 and c, it is considered accurate. It can not therefore be defined more accurately than the equation **(77)**.

2.15. The Planck Surface s_p

The Planck surface s_p is defined by:

$$s_p = l_p^2 = c^2 \cdot t_p^2 = \frac{\hbar \cdot G}{c^3} \approx 2.61186(31) \times 10^{-70} \mathrm{m}^2 \quad \mathbf{(78)}$$

According to the CODATA 2010 [1], the value of the Planck length is given by $l_p \approx 1.616199(97) \times 10^{-35}$ m.

In the equation **(78)**, the Planck length l_p represents the smallest length unit that a photon may pass while being measurable. Therefore, the Planck surface is the smallest measurable unit of area.

It is interesting to note, without trying to demonstrate it here, that the Planck surface s_p can be described in terms of the apparent radius of curvature of the universe R_u and fine structure constant α:

$$s_p = R_u^2 \cdot \alpha^{57} = \frac{c^2}{H_0^2} \cdot \alpha^{57} \quad \mathbf{(79)}$$

For a better accuracy in the evaluation of the Planck surface s_p, let's try to express the equation **(78)** without using the gravitational constant G and without the Planck constant h.

Using the equations **(14)**, **(15)** and **(16)** in the equation **(78)**, we get:

$$s_p = \frac{r_e^2 \cdot \alpha^{19}}{\beta} \approx 2.611861426(17) \times 10^{-70} \mathrm{m}^2 \quad \mathbf{(80)}$$

2.16. The Planck Volume v_p

The Planck volume v_p is defined by:

$$v_p = l_p^3 = c^3 \cdot t_p^3 = \left(\frac{\hbar \cdot G}{c^3}\right)^{3/2} \approx 4.22110(76) \times 10^{-105} \mathrm{m}^3 \quad \mathbf{(81)}$$

According to the CODATA 2010 [1], the value of the Planck length is given by $l_p \approx 1.616199(97) \times 10^{-35}$ m.

In equation **(81)**, the Planck length l_p is the smallest unit of length that a photon may pass while being measurable. Therefore, the Planck volume represents the smallest unit of measurable volume.

It is interesting to note, without trying to demonstrate it here, that the Planck volume v_p may be described as a function of the apparent radius of curvature of the universe R_u and the fine structure constant α:

$$v_p = \left(R_u^2 \cdot \alpha^{57}\right)^{3/2} = \left(\frac{c^2}{H_0^2} \cdot \alpha^{57}\right)^{3/2} \tag{82}$$

For a better accuracy in measuring the Planck volume v_p , let's try to express the equation **(81)** without using the gravitational constant G and without the Planck constant h.

Using the equations **(14)**, **(15)** and **(16)** in the equation **(81)**, we get:

$$v_p = \left(\frac{r_e^2 \cdot \alpha^{19}}{\beta}\right)^{3/2} \approx 4.221095685(41) \times 10^{-105} \text{m}^3 \tag{83}$$

3. CONCLUSION

The Planck units that we have listed all correspond to a physical limit or a parameter that is optimized.

We have listed several equations showing a close relationship between the Planck units and the different parameters of our universe whose constant β that comes directly from our model of the universe [10]. Without this constant, we probably could not find more accurate equations for Planck units than the existing one.

Thanks to our work on the universal gravitational constant G, it is possible to improve the value of certain Planck units by redefining them based on the characteristics of the electron and fine structure constant α.

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