Theoretical Calculation of the Cosmological Constant and of the Density of « Dark Energy » in the Universe

Claude Mercier eng., December 25, 2018 claude.mercier@cima.ca
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In 1916 Einstein published his first paper on the theory of general relativity. The equations of this theory predicted that the universe was either shrinking or expanding.

According to the ideas conveyed at that time, the universe was static. Einstein adhering to this idea tried to modify his equation to make his cosmological model static. In 1917, he published this new model where, for the first time, appears this famous constant.

In 1929, thanks to his observations, Hubble showed that the universe was rather expanding [4]. Of course, Einstein bitterly regretted having modified his equation because he would have been the first to predict the expansion of the universe. Einstein stopped using his cosmological constant after realizing that the universe was not static.

Nowadays, several astrophysicists are trying to revive the usefulness of the cosmological constant to explain the expansion of the universe, including the hypothesis of the presence of dark energy. Unfortunately, no one has been able, until now, to determine the real nature of this famous energy.

In order to determine the value of the cosmological constant, we will show that dark energy is actually composed of photons of different wavelengths. In the vast majority of cases, these wavelengths are invisible to us because we are not able to receive them. Consequently, this energy, which we see the effects, seems mysterious to us, without really being it.

KEY WORDS: Cosmological constant, dark energy, relativity

1. INTRODUCTION

Einstein's equations show that the universe is either expanding or contracting. For other considerations [22], such as the third law of thermodynamics, the universe can not evolve by contracting.

According to our model of the universe [16], the "big crunch" will never take place since the universe extends against a total absence of radiation pressure [22]. Moreover, according to our model, gravitation is a phenomenon that occurs when we are inside the universe and is due to a radiation pressure that pushes objects against each other. For these reasons, the universe can only expand.

Whatever the true nature of "dark energy," the cosmological constant represents the missing piece to ensure that Einstein's general relativity field equations can represent a model of a static universe.

Today, because of Hubble's research [4], the vast majority of astrophysicists agree that the universe is expanding.

In this article, we will show, thanks to our model of the universe, that it is possible to theoretically determine the value of the cosmological constant and, by the same token, the energy density of the vacuum. We will then show that it is not necessary to make a concept of "dark energy", but that this energy is simply due to a bath of photons of different wavelengths, for the most part, undetectable due to of our limited capabilities to make the right sensors.

2. VALUE OF THE PHYSICS PARAMETERS USED

Let us outline all the basic physics parameters we intend to use in this article. These values are all available in CODATA 2014 [1].

- Speed of light in vacuum $c \approx 299792458$ m/s
- Planck length $L_p \approx 1{,}616229(38) \times 10^{-35}$ m
- Planck mass $m_p \approx 2{,}176\,470(51) \times 10^{-8}$ kg
- Masse of the electron $m_e \approx 9{,}10938356(11) \times 10^{-31}$ kg
- Classical radius of the electron $r_e \approx 2{,}8179403227(19) \times 10^{-15}$ m
- Fine structure constant $\alpha \approx 7{,}2973525664(17) \times 10^{-3}$
- Universal gravitational constant $G \approx 6{,}67408(31) \times 10^{-8}$ m^3/(kg·s^2)
- Planck constant $h \approx 1{,}054571800(13) \times 10^{-8}$ J·s

3. THE FIELD EQUATIONS OF GENERAL RELATIVITY

As we have already explained [21], the "gravitational force" as Newton had imagined it, is only a concept where a force is associated with a mass that accelerates. Newton's gravitation equation is not an explanation of gravitation, but a quantification method.

On his side, Einstein had a different conception of gravitational force. He made a thought experiment involving a man located in an elevator that is located in the sidereal vacuum (out of gravity). It shows that a pull on the elevator will give the observer, located inside the cabin, a feeling of a gravitational force. In fact, he will not be able to tell the difference between a gravitational force and an accelerated traction.

For Einstein, the universe is bent by the presence of masses and objects follow these curved paths. Einstein's 1917 field equations of general relativity [18] includes a cosmological constant Λ which he voluntarily added to his equations in order to conform to the idea of the time that the universe was static [17, 23]:

$$R_{\mu\nu} - \frac{1}{2} R \cdot g_{\mu\nu} + \Lambda \cdot g_{\mu\nu} = \frac{8\pi \cdot G}{c^4} \cdot T_{\mu\nu} \quad \textbf{(1)}$$

After the Hubble's discovery of the expansion of the universe in 1929 [4], Einstein dropped the idea of using a cosmological constant.

In equation (1), $R_{\mu\nu}$ and R are the Ricci tensor and scalar respectively and g_{uv} is the tensor that describes the space-time structure in a 4-dimensional space (length, width, height, time). G represents the Newton's universal gravitational constant and *c* is the speed of light in vacuum (and we add, out of gravitation).

As Hubble found through his observations in 1929, the universe is expanding [4]. This corresponds to a non-zero and positive value of cosmological constant Λ. The Λ constant represents, in a certain way, the energy density of space.

The value of the cosmological constant Λ is given by [23]:

$$\Lambda = \frac{8\pi \cdot G \cdot \rho_v}{c^2} \quad \textbf{(2)}$$

In this equation, ρ_v represents the density of the vacuum (volumetric mass of the vacuum). It is here that many physicists involve the idea of a "dark energy" since they believe, wrongly, that vacuum is an absence of everything. For that reason, we do not really like this interpellation since it has a connotation that leads us to believe that vacuum is an absence of everything. It is an absence of matter, but, in reality, vacuum is full of photons of different wavelengths.

Since the parameter ρ_v is required to determine the value of the cosmological constant, we will focus on calculating its value.

4. CALCULATION OF THE COSMOLOGICAL CONSTANT
4.1. The Starting Equations

To explain the energy contained in the space vacuum of our universe, some will involve the concept of "dark energy".

According to our model of the universe, "dark energy" would in fact consist only of photons of wavelengths varying from $2\pi \cdot L_p$ up to $2\pi \cdot R_u$. The L_p value represents the Planck length ($\sim 10^{-35}$ m) and R_u is the apparent radius of curvature of the luminous universe ($\sim 10^{26}$ m).

Why would we call this "dark energy"? Simply because, up to now, it is impossible to make antennas capable of receiving these wavelengths and detecting them. Our world on Earth is very limited with respect to the dimensions of our universe (as much in the infinitely large as and in the infinitely small). For a wavelength to be sensed, an antenna having a length equal to a quarter of a wavelength must be manufactured.

The part of the electromagnetic spectrum that is presently detectable represents a very small part of the real spectrum of electromagnetic waves. But our inability to detect these electromagnetic waves does not mean they do not exist. They are there and they influence us by creating a radiation pressure.

Remember that not so long ago, electromagnetic waves were unknown to us. They were predicted by James Clerk Maxwell in 1864. Radio waves were discovered in the late nineteenth century with the works of Alexander Popov, Heinrich Hertz, Edward Branly and Nikola Tesla.

In our opinion, that the universe is bent by the masses and that the equations of general relativity can precisely determine the trajectories of objects do not represent in themselves a true explanation of the nature of the forces that we can associate them with.

In our opinion, the true nature of the "gravitational force" comes more from radiation pressure of photons of different wavelengths [21]. As long as the observer is inside the universe, he will observe an acceleration of the objects toward each other that may give him the feeling that the forces exist. But, on the border limit of our universe, it is expanding against a total absence of thermodynamic radiation pressure [22]. The universe has no choice but to extend forever, as long as it does not encounter any reverse radiation pressure.

The best analogy to understand this concept is Archimedes' principle. As long as we are under water, we are submitted to water pressure. Once out of the water, the pressure stops (if we do not consider the air pressure).

Knowing that "dark energy" can be associated with the presence of different wavelengths photons in space, let us determine the percentage of the apparent mass of the universe Ω_v that can be associated with the energy of so-called photons in the interstellar vacuum.

Let us consider that the total mass of the universe m_t can have two provenances:

1) m_Λ: The mass associated with the energy of photons;
2) m_m: The mass associated with matter (whatever it may be, that is, "dark matter," baryon matter, etc.).

As an observer located on the so-called mass of the expanding material universe, we measure its mass at rest because we travel at the same speed as this one. Consider then that m_m is the mass at rest of matter.

The total mass is of course the apparent mass of the universe m_u and this can be obtained by summing the masses coming from the energy associated with the photons and from the matter (measured "at rest"):

$$m_u = m_\Lambda + m_m \tag{3}$$

Thanks to our previous research [15], we know how to precisely evaluate the value of the apparent mass of the universe m_u from the universal gravitational constant G and the Hubble constant H_0:

$$m_u = \frac{c^3}{G \cdot H_0} \approx 1.728 \times 10^{53}\ \text{kg} \tag{4}$$

This equation is identical to that obtained by Carvalho [14].

Other works [3] have also enabled us to calculate precisely the gravitational constant G as a function of the speed of light in vacuum c (out of gravitation), of the fine structure constant α, of β, the mass of electron m_e and the classical radius of the electron r_e.

$$G = \frac{c^2 \cdot r_e \cdot \alpha^{20}}{m_e \cdot \beta} \approx 6.673229809(86)\ \frac{\mathrm{m}^3}{\mathrm{kg} \cdot \mathrm{s}^2} \tag{5}$$

Let us note that this value is very similar to that obtained in CODATA 2014 which is $G \approx 6.67408(31) \times 10^{-11}$ m^3/(kg·s^2). We consider that equation (5) is more precise and it will be used in this document.

These same research works [3] allowed us to precisely calculate the Hubble constant H_0 as a function of the speed of light in vacuum c (out of gravitation), the fine structure constant α, β, of the classical radius the electron r_e.

$$H_0 = \frac{c \cdot \alpha^{19} \cdot \beta^{1/2}}{r_e} \approx 72.09548580(32)\ \frac{\mathrm{km}}{\mathrm{s} \cdot \mathrm{MParsec}} \tag{6}$$

This result is similar to the Salvatelli value [2] which is $72.1^{+3.2}_{-2.3}$ km/(s·MParsec). We consider that equation (6) is more precise and it will be used in this document.

According to Hubble, the universe is expanding [4]. But if we go back in time, our universe necessarily comes from a denser point. If the universe is born of a singularity during the big bang, the initial momentum was necessarily zero since it could not have any movement with respect to any reference.

Whatever the momentum of photons and matter are, the total sum p_t of the momentum is zero. This finding will constitute a second equation:

$$\vec{p}_t = \vec{p}_\Lambda + \vec{p}_m = 0 \tag{7}$$

Thanks to equations (2) and (7), we are able to determine the proportions of masses that are required. We now have to find the values of p_Λ and of p_m to complete the equation.

Be careful, to evaluate the momentum, we will have to do it as an observer located at rest in the center of mass of the universe. This has the consequence that we will have to evaluate the momentum taking into account the relativistic effects due to the speed of the expanding universe.

3.1 Calculation of Photons Momentum p_Λ

Let us determine the required equation to describe p_Λ in equation (7).

The momentum of each individual photon and each particle of matter are vectorial and may have any direction in space. But if all the modules of these vectors were aligned on the same line (taking into account the direction + or -), we would always have the following equality:

$$\left|\vec{p}_{\Lambda}\right| = \left|\bar{p}_{m}\right| \tag{8}$$

In a way, it's not too difficult to understand. Each time a photon is emitted, it transmits to the matter the same momentum (inverse). If we no longer take into account individual vector directions of each photon and particle of matter, global equality (only) still holds because the sum must be zero. Let us simplify the notation of equation (13) to keep only scalar values. So we have:

$$p_{\Lambda} = p_{m} \tag{9}$$

The momentum p of a photon is normally determined by Planck constant h and its wavelength λ:

$$p = \frac{h}{\lambda} \tag{10}$$

Due to the corpuscular and wave duality, we know that the energy of an electromagnetic wavelength λ may be associated with the energy contained in a mass m. Thanks to Einstein's energy equation (left, in equation (11)) and Planck's energy equation (on the right, in equation (11)), we obtain the following equality:

$$m_{\lambda} \cdot c^{2} = \frac{h \cdot c}{\lambda} \tag{11}$$

Thanks to equations (10) and (11), we are able to obtain the following equation which gives the momentum as a function of the mass m_λ associated with the photon of wavelength λ:

$$p = m_{\lambda} \cdot c \tag{12}$$

Of course, the value of photon mass m_λ depends on the electromagnetic wavelength. In space vacuum, several wavelengths may be present varying from $2\pi \cdot L_p$ up to $2\pi \cdot R_u$. Let us suppose there can be n_λ photons for each wavelength λ. Let us sum the momentum p_Λ of all the photons in the universe, from $\lambda = 2\pi \cdot L_p$ up to $\lambda = 2\pi \cdot R_u$, by discrete hops of $i \cdot L_p$ (where i = 1 to $N^{1/2}$) :

$$p_\Lambda = m_\Lambda \cdot c = \sum_{i=1}^{i=\sqrt{N}} n_\lambda \cdot m_\lambda \cdot c \Big|_{\lambda = 2\pi L_p \cdot i} \quad \textbf{(13)}$$

Here we have associated a mass value m_Λ to all of the photons in the universe. Let us note that N is the maximum number of photons of wavelength $2\pi \cdot R_u$ that can be contained in the mass of the universe m_u (by associating a mass with the energy of each photon). Its value comes from the hypothesis on large numbers of Dirac [7]. We have shown several methods to obtain it [8], but here is the most precise one [3]:

$$N = \frac{1}{\alpha^{57}} \approx 6.3 \times 10^{121} \quad \textbf{(14)}$$

In this equation, α is the fine structure constant.

In equation (13), let us note that Planck length L_p multiplied by the square root of N gives precisely the value of the apparent curvature radius of the universe R_u [6, 9 , 10] :

$$L_p \cdot \sqrt{N} = R_u \quad \textbf{(15)}$$

3.2 Calculation of the Momentum of the Matter p_m

Let us determine the necessary equation to describe p_m in equation **(7)**.

Let us start from our model of the universe. The universe is expanding. Globally, light and matter move away from a center of mass. But matter can not move as fast as light. Finally, we end up with a first sphere of matter (which we called the material universe) nested in another, larger sphere, which contains electromagnetic waves (which we called the luminous universe).

Here we make no distinction between matter types (baryonic or "dark matter"). We look at it as a whole. Because of Einstein's principle of relativity, we know that matter can not move from the center of mass as fast as light. We determined by calculations (thanks to the resolution of a 4 equations and 4 unknowns system [16]) that the matter moves at $\beta \cdot c$. Here, the value of β is given by:

$$\beta = 3 - \sqrt{5} \approx 0.764 \quad \textbf{(16)}$$

We also determined that the luminous universe was spinning on itself at a speed close to that of light. By the same fact, the tangential velocity of the material universe must also be $\beta \cdot c$. We note here that each particle of matter moves in 2 directions at a time. We can not simply do a vector summation of velocities because the velocities involved here are relativistic in nature. We must therefore make a relativistic vector summation.

Let us show how to make a relativistic summation of two velocity vectors $u_{x,y,z}$ and $w_{x,y,z}$. To simplify the problem, we choose a frame of reference to express the velocity vector $w_{x,y,z}$ so that $w_y = 0$ and $w_z = 0$. It is a question of making the rotations and the translations which are necessary for superimpose the vector $w_{x,y,z}$ with the abscissa axis. It is always possible to do it.

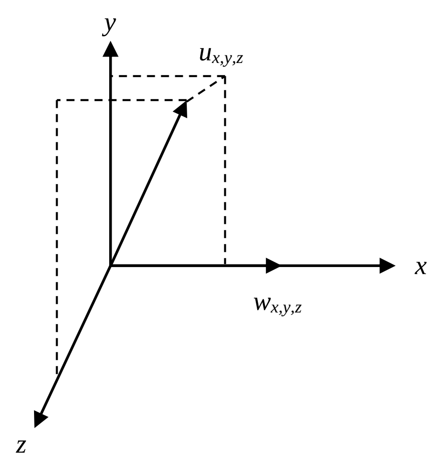

Figure A

It gives the following three equations for the resulting vector $v_{x,y,z}$.

$$v_x = \frac{u_x + w_x}{1 + \frac{u_x \cdot w_x}{c^2}} \tag{17}$$

$$v_y = \frac{u_y \sqrt{1 - \left(\frac{w_x}{c}\right)^2}}{1 + \frac{u_x \cdot w_x}{c^2}} \tag{18}$$

$$v_z = \frac{u_z \sqrt{1 - \left(\frac{w_x}{c}\right)^2}}{1 + \frac{u_x \cdot w_x}{c^2}} \tag{19}$$

Let us suppose now that, locally, we simultaneously analyze the expansion and the rotation of the material universe. Let us consider the case where the material universe is expanding with a $\beta{\cdot}c$ speed on the y-axis, that is, $u_x = 0$, $u_y = \beta{\cdot}c$ and $u_z = 0$. Let us suppose arbitrarily, the universe is also rotating on the on the x-axis with a tangential speed $w_x = \beta{\cdot}c$ with $w_y = 0$ and $w_z = 0$ (as previously explained).

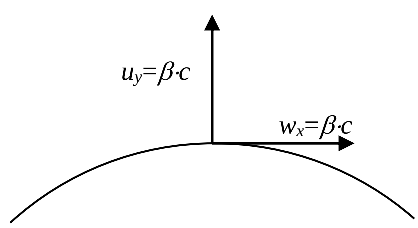

Figure B

Let us apply this new data to equations (17), (18) and (19). These equations are simplified to obtain:

$$v_x = \beta \cdot c \tag{20}$$

$$v_y = \beta \cdot c\sqrt{1 - \beta^2} \tag{21}$$

$$v_z = 0\,\text{m/s} \tag{22}$$

If we calculate the module $\left|v_{x,y,z}\right|$ of the resulting vector, we obtain:

$$\left|v_{x,y,z}\right| = \sqrt{v_x^2 + v_y^2 + v_z^2} = c \qquad \textbf{(23)}$$

By making the necessary replacements, we obtain:

$$v = \left|v_{x,y,z}\right| = \beta \cdot c\sqrt{2-\beta^2} \approx 0.909 \cdot c \qquad \textbf{(24)}$$

Matter, as a whole, travels at a relativistic speed. Thanks to his special relativity equations [5], Einstein has shown that a rest mass m_m accelerated at speed v is perceived by an observer at rest as being a mass equal to m':

$$m' = \frac{m_m}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \textbf{(25)}$$

The momentum of a relativistic mass m' is given by:

$$p_m = m' \cdot v \qquad \textbf{(26)}$$

Suppose its rest mass is mm. Using equations (24) and (25) in equation (25), its momentum, at velocity v, becomes:

$$p_m = \frac{m_m \cdot \beta \cdot c\sqrt{2-\beta^2}}{\sqrt{1-\beta^2 \cdot \left(2-\beta^2\right)}} \qquad \textbf{(27)}$$

3.3 Resolution of a system of 2 equations and 2 unknowns

Coming from equation **(3)**, we obtain this first equation:

$$m_u = m_\Lambda + m_m \qquad \textbf{(28)}$$

Using equations **(9)**, **(13)**, and **(27)** we obtain this second equation:

$$m_\Lambda \cdot c = \frac{m_m \cdot \beta \cdot c\sqrt{2-\beta^2}}{\sqrt{1-\beta^2 \cdot \left(2-\beta^2\right)}} \qquad \textbf{(29)}$$

The two unknowns of this system of equations are m_Λ and m_m. With 2 equations and 2 unknowns, we can solve this system of equations.

In equation (28), Let us divide all terms by m_u to obtain:

$$1 = \frac{m_\Lambda}{m_u} + \frac{m_m}{m_u} \qquad (30)$$

Let us define Ω_Λ as the ratio of the mass associated with photons in the universe to the mass of the universe m_u:

$$\Omega_\Lambda \equiv \frac{m_\Lambda}{m_u} \qquad (31)$$

Let us define Ω_m as the ratio of the mass associated with matter (dark and baryonic) in the universe with respect to the mass of the universe m_u:

$$\Omega_m \equiv \frac{m_m}{m_u} \qquad (32)$$

We then obtain the following equation:

$$1 = \Omega_\Lambda + \Omega_m \qquad (33)$$

From equation (29), we replace the value of mm with equation (28) to obtain:

$$\Omega_m = \frac{m_m}{m_u} = \frac{1}{\frac{\beta \cdot \sqrt{2-\beta^2}}{\sqrt{1-\beta^2 \cdot \left(2-\beta^2\right)}}+1} \approx 0.3141 \qquad (34)$$

Similarly, from equations (33) and (34)**,** we obtain:

$$\Omega_\Lambda = \frac{m_\Lambda}{m_u} = 1 - \frac{1}{\frac{\beta \cdot \sqrt{2-\beta^2}}{\sqrt{1-\beta^2 \cdot \left(2-\beta^2\right)}}+1} \approx 0.68586879.. \qquad (35)$$

The mass of the universe would consist of 68.59% of different wavelengths photons and 31.41% of matter ("dark matter" and baryon).

Let us note that equations (34) and (35) give precise numbers that are truly constant since they depend only on β which is also constant over time.

In 2018, Planck probe from the European Space Agency (ESA) measured [20]:
Ω_Λ = 68.97 % ±0.57 and Ω_m = 31.03 % ±0.57.

In 2003, SDSS and WMAP probes from NASA measured the following values [24]:
Ω_Λ = 68.5 % +3.2/-4.1 and Ω_m = 31.5 % +4.1/-3.2.

In 2013, the WMAP probe from NASA in combination with the CMB obtained the following values [26]: Ω_Λ = 72.7 % ±3.8 and Ω_m = 27.3 % ±4.9.

We find that our theoretical results are consistent with the measurements.

3.4 Calculation of the Cosmological Constant

We now want to calculate the cosmological constant.

The equation of the mass density of photons in the universe is given by ρ_Λ :

$$\rho_\Lambda = \frac{\Omega_\Lambda \cdot H_0^2}{8\pi \cdot G} \approx 2.232 \times 10^{-27}\ \text{kg/m}^3 \tag{36}$$

The cosmological constant varies as a function of ρ_Λ [23]:

$$\Lambda = \frac{8\pi \cdot G \cdot \rho_\Lambda}{c^2} \tag{37}$$

Using these last two relationships, we get:

$$\Lambda = \frac{\Omega_\Lambda \cdot H_0^2}{c^2} = \frac{\Omega_\Lambda}{R_u^2} \approx 4.166 \times 10^{-53}\ \text{m}^{-2} \tag{38}$$

We find that the cosmological "constant" Λ is not a constant since it varies as a function of the apparent radius of curvature of the universe R_u.

It can be shown that the following equation is also true because the apparent radius of curvature of the R_u universe can be described as a function of the Planck length and the constant N [8]:

$$\Lambda = \frac{\Omega_\Lambda}{N \cdot L_p^2} \approx 4.166 \times 10^{-53}\ \text{m}^{-2} \tag{39}$$

The value of the cosmological constant is almost zero compared to the other terms of equation (1) . By approximation, we obtain [19, 25]:

$$R_{\mu\nu} - \frac{1}{2} R \cdot g_{\mu\nu} = \frac{8\pi \cdot G}{c^4} \cdot T_{\mu\nu} \tag{40}$$

5. CONCLUSION

We are able to theoretically calculate the value of the cosmological constant as well as the mass ratio of the energy of photons contained in the universe without having to involve the concept of dark energy. In fact, "dark energy", as some call it, is nothing other than whole electromagnetic waves contained in the universe. As we are not yet able to sense the different wavelengths involved, this energy seems impalpable and mysterious.

We note that our theoretical calculations lead us to similar quantities (for Ω_Λ and Ω_m) to those obtained by the European Space Agency (ESA) thanks to the Planck probe mission [20] and thanks to the SDSS and WMAP probes from NASA [24, 26].

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