Calculation of the Apparent Radius of Curvature of the Universe

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Since the beginning of its existence, the universe is in expansion [1]. *It expands at the speed of light* [2], *at least for its luminous part. If the universe is a large sphere expanding at the speed of light in vacuum c, its apparent radius R_u of curvature can easily be deduced* [3] *by the product of the speed c and the apparent age of the universe (that corresponds to the inverse of the Hubble constant H_0)* [4].

The calculated value does not necessarily correspond to the reality. Effectively, the notion of ″radius of curvature″ that we use is local. If the universe is spherical, that radius corresponds to the radius of the sphere. If the universe has a more complex morphology (e.g. a peanut or a tore shape), the radius of curvature corresponds therefore to a local notion that we try to associate with a sphere and cannot be extended to the rest of the universe. The calculated parameter corresponds however to a certain reality that might be associated with our universe, at least locally.

The fact that we qualify the radius R_u of curvature of the universe as ″apparent″ refers to the fact that the calculated radius is true only for the actual speed of light c. Since we claim that light accelerates over the time because of the expansion of the universe, it is possible that the true distance covered since its creation might be different from the calculated one. The ″apparent″ radius of curvature that we calculate here is equivalent to finding the radius of curvature of a sphere that would be in expansion at a constant speed equal to the actual speed of light c.

In this document, we will propose different methods to calculate R_u. One of the best method that we will find will show that there is a close relation between the infinitely big (the universe) and the infinitely small (at the electron level). However, the best precision is obtained using the Rydberg constant R_∞. With this constant, we obtain the following value: $R_u \approx 1.2831078806 \pm 0.0000000068 \times 10^{26}$ m.

Thanks to the Einstein laws of relativity, we know that it is impossible for the material universe to be in expansion at the speed of light c, if not, the required energy would be infinite. We will calculate the apparent radius r_u of curvature which corresponds to our actual apparent position with respect to the apparent origin of the expansion sphere of the universe. We will show that $r_u \approx 9.802071983 \pm 0.000000052 \times 10^{25}$ m.

Furthermore, we will also calculate the apparent radius of curvature of the horizon [3] *of the universe* $r_h \approx 6.058013646 \pm 0.000000032 \times 10^{25}$ m.

KEY WORDS: Radius, Hubble, universe, electron, Rydberg

1. INTRODUCTION

Let's begin by saying that the term "apparent radius of curvature of the universe" is proper to the notions that we want to discuss in this article. Certain authors will refer to this same value using different names: "scale factor" [17], "Hubble radius" [18], "radius of the universe" [19], "size of the universe" [20], "curving radius of the space-time" [20] and many others.

Considering that the limits of the luminous universe are determined by the all areas where there is a presence of photons, the limits of the universe are in expansion. The universe is not static as Einstein wished to understand it in his cosmology of the universe [5]. Hubble noted, by observation, that the universe is expanding [1]. If we model the luminous universe as being a sphere in expansion, its apparent radius of curvature is R_u , at least locally.

Even if the luminous universe is in expansion at the speed of light c, the material universe cannot expand at that speed. According to a model of the universe that we presented in the past [3], we came to the conclusion that the material universe should expand at a speed of $\beta \cdot c \approx 0.76 \cdot c$. Its apparent radius of curvature is necessarily smaller than R_u.

Since light accelerates over time [3], it is possible to find a moment in the history of the universe when the speed of light is null. At that moment, the apparent curving radius of the universe was equal to r_h. That radius is the one of the horizon of the universe.

The origin of the three spheres (of the luminous universe, of the material and the horizon of the universe) are superposed. The sphere of the material universe is included in the one of the luminous universe. The sphere of the horizon inbricated in the two preceding ones [3].

In this article, we want to calculate precisely the apparent values of the radiuses of curvature R_u, r_u and r_h. To do so, we will begin by recalling the values of the Hubble constant H_0 and of the universal gravitational constant G that we have calculated in preceding articles [6]. Using these parameters, we will calculate the apparent radius of curvature of the luminous universe using different ways while evaluating the precision of the shown results. This will allow us to evaluate which method is the most precise one among those presented. From this method, we will calculate the value of the apparent radius of curvature of the universe r_u at our actual position in the universe and the value of the apparent radius of curvature of the horizon of the universe r_h.

Let us recall that the precise values of H_0 and G used in this article could have been tested by using them in a theoretical calculation of the mass of the electron m_e [7]. This calculation is based on an improved version (thanks to the value of the constant β, calculated in our article on the acceleration of light [3], and to the fine structure constant α) of the empirical equation stated by Weinberg [15,16]. Having obtained a theoretical value that is identical with the one mentioned in the worldwide reference to the physics constants which is the CODATA [8], we conclude that they are more precise than the H_0 and G values that are recommended until now in the CODATA (for G) and by all the results published to this day (for the value of H_0).

Some will claim that it might be utopian to calculate the theoretical values of R_u, r_u and r_h with so much precision. However, as the knowledge of the human beings improve, the values of the different parameters of the universe become more precise. As long and as much as the observed values agree with the theoretical values shown here, we will conclude that our model of the universe agrees. The day when the two values diverge from the incertitudes, our model will have to be reviewed and perfected.

2. VALUES OF SOME USEFUL PARAMETERS

2.1. Theoretical Hubble Constant H_0 from Previous Works

Many research teams throughout the world have developed their own way of measuring the Hubble constant and obtain results that they hope to be as precise as possible. Looking backward, we see that some results are probably shown with error margins that do not match up. Since we do not know all the details that led to these results, it becomes difficult to give more credit to the one or the other of these measuring methods.

In previous works [6,7] that we made available on Internet, we showed that the value of the Hubble constant H_0 could be given by an equation with a precision that depended mainly on the speed of light c, on the fine structure constant α and on the classical radius of the electron r_e.

$$H_0 = \frac{c \cdot \alpha^{19} \cdot \beta^{1/2}}{r_e} \tag{1}$$

$$H_0 \approx 72.09548632 \pm 0.00000046 \text{km}/(s \cdot MPar\text{sec}) \quad \textbf{(2)}$$

According to the CODATA 2010 [8]:

- Actual speed of light in vacuum $c \approx 299792458$ m/s
- Fine structure constant $\alpha \approx 7.2973525698 \pm 0.0000000024 \times 10^{-3}$
- Classical electron radius $r_e \approx 2.8179403267 \pm 0.0000000027 \times 10^{-15}$ m

The value of β is a pure number. It gives the ratio between the expansion speed of the material universe and the speed of light in vacuum c [3] :

$$\beta = 3 - \sqrt{5} \approx 0.764 \quad \textbf{(3)}$$

Our method to obtain H_0 does not come from direct measurements [6,7]. It supposes, among others, that there is a theoretical link between this parameter and the constants that are used in the equation **(1)**.

Since the calculations of the present document are based on equations that depend on H_0, the precision of this parameter looks crucial to us. If all hypotheses that we made in the past are true, it is logical to use these results in our calculations… until we are confronted by a phenomenon that infirm what we found.

Since we want to show values that agree with independent researches, let's note that the value of H_0 obtained in **(2)** agrees with the one measured by Xiaofeng Wang [9] and his team who obtained $H_0 \approx 72.1 \pm 0.9$ km/(s·MParsec).

2.2. Theoretical Universal Gravitational Constant *G* from Previous Works

Since certain calculations of this present document are based on equations that depend on G, the precision of this parameter looks crucial to us. As for the Hubble constant, it is logical for us to use the results of calculations that we obtained from previous works [6]. Consequently, the universal gravitational constant G can be described with great precision by using the following equation that mainly depends on the fine structure constant α , on the classical radius of the electron r_e , on its mass m_e and on the speed of light in vacuum c:

$$G = \frac{c^2 \cdot r_e \cdot \alpha^{20}}{m_e \cdot \beta} \approx 6{,}67323036 \pm 0{,}0000003 \times 10^{-11} \text{ m}^3/\left(kg \cdot s^2\right) \quad \textbf{(4)}$$

According to the CODATA 2010 [8]:

- Universal gravitational constant $G \approx 6.67384 \pm 0.00080 \times 10^{-11}$ m^3/(kg·s^2)
- The rest mass of the electron $m_e \approx 9.10938291 \pm 0.00000040 \times 10^{-31}$ kg

Let's note that the value of the universal gravitational constant G obtained in the equation **(4)** agrees up with the one mentioned in the CODATA 2010 [8]. Since we claim that the value of equation **(4)** is more precise than the one of the CODATA, we will use with advantage this value for the remainder of the present article. Effectively, it will allow us to conclude, at the end, that we can calculate precisely the mass of the electron, something that would have been impossible with the value of G that comes from the CODATA [8].

3. APPARENT RADIUS OF CURVATURE THE UNIVERSE R_u

3.1. Signification of the Term «Apparent Curving Radius »

Some will be tempted to believe that our paper suggests that the universe is a sphere of radius R_u. This is not totally true. We want to bring an important nuance. Whatever be the reality of things in the universe, there seems to be a local space curvature which leads us to believe that the universe is spherical with a radius R_u. But this is not necessarily the case. It is for this reason that we add, out of prudence, the term « apparent ». The universe could very well not be spherical and have, for example, the shape of a peanut, of a tore or something else. But locally, the space seams curved in such a way that it leads us to think that it is spherical and of radius R_u. The value of R_u can be seen as a characteristic parameter of the universe that is useful to describe the local radius of curvature of the universe.

3.2. Homogeneity and Isotropy Hypothesis

At the risk of repeating ourselves, it is very well possible that R_u does not represent the reality. The reality is complex in its details. When we speak about the universe as a whole, we most certainly cannot talk about the local particularities of space. We have to treat the whole as a homogeneous and isotropic mix.

Without being conscious of it, our macroscopic calculations are the statistical results of chaotic microscopic phenomena. For example, the universal gravitational law of Newton can be deduced from the statistical laws of Poisson [10]. With the microscopic scale, it is impossible to know with certitude the behaviour of particles and events. Only the statistical results are known. In the same manner, at the level of the solar and galactic system, space does not seem to be homogeneous. But, on the scale of the universe, space can be statistically treated as a homogeneous mixture.

This being said, in this document, we try to determine different methods to quantify the value of the apparent radius of curvature R_u. Among these methods, we will try to identify the one that seems to be the most precise one as a function of the incertitude calculation linked to each equation.

3.3. Calculation of Error

To know the value of the incertitude linked to the different equations, we will use the square root of the sum of the squared deviations between the calculations of R_u using the values of the CODATA 2010 [8] and the calculations of R_u using in turn the values added with the absolute errors of each constant.

For example, let's suppose the case where R_u varies as a function of r_e, β and α as we shall soon see in the equation **(15)**. The value of β is an exact irrational number. So, there is no error in this value. Therefore, we will make the following calculation assuming that Δr_e and $\Delta\alpha$ are the absolute value of the errors bearing on r_e and α respectively.

$$\text{Error} = \sqrt{\left(R_u(r_e,\alpha) - R_u(r_e + \Delta r_e,\alpha)\right)^2 + \left(R_u(r_e,\alpha) - R_u(r_e,\alpha + \Delta\alpha)\right)^2} \qquad \textbf{(5)}$$

3.4. Value of R_u that Comes from a Constant Speed of Light over Time

Einstein postulated that the speed of light was always the same everywhere [11,14]. According to us, he did not have the tools to verify if his postulate was valid over time. Moreover, even with the actual technology, many are still convinced that the speed of light is constant over time. As a proof, the CODATA 2010 considers the value of c as being "exact" [8]. Let's simply say that many wish that it be constant for its non-constancy would put to question many elements of physics acquired through experience, including all the units of measurement which are based on the speed of light. From our point of view, we are convinced that light slowly accelerates over time [3]. The acceleration would be so small that it is beyond the detectable threshold of contemporary instruments (an increase of 1 m/s every 35.4 years). However, by carrying out many experiments through many decades, it is possible to see the cumulative effect of the acceleration of light over years. The Pioneer acceleration is a good example of it [3]. This phenomenon comes from the fact that continuing to believe that the speed of light is constant over time, we interpret the frequency variations in the

Doppler Effect as being a deceleration of the Pioneer probes. In facts, the probes do not slow down. It is the light that accelerates.

In the present article, we are searching for the apparent radius of curvature of the universe. Obviously, since the universe is in expansion, this parameter changes over time. However, using the actual speed of light in vacuum and the actual **apparent age** of the universe, it is possible to obtain the actual apparent radius of curvature of the universe. This amount to doing everything as if the speed of light would have always been constant over time during a period equivalent to the apparent age of the universe.

The apparent age of the universe is given by the following equation:

$$T_u = \frac{1}{H_0} \qquad \textbf{(6)}$$

Consequently, starting supposedly from a point of singularity (coming from the big bang theory) light would have had the time to travel the following distance:

$$R_u = c \cdot T_u = \frac{c}{H_0} \approx 1.2831078807 \pm 0.0000000083 \times 10^{26}\,\mathrm{m} \qquad \textbf{(7)}$$

It is an easy way to calculate the apparent radius of the universe. This way is somewhat like a tangent to the curve. It implies that the speed of light in vacuum has always been the same and equal to c. Besides, that is why we would like to mention, once again, that the R_u value is an apparent radius of curvature.

This presentation has the advantage of being very concise. However, unless we use our Hubble constant described in **(2)**, which has high accuracy, this presentation has the disadvantage of using H_0 which is not considered to be a fundamental constant of physics.

3.5. Value of R_u as a Function of the Classical Radius of the Electron

Now, let's try to establish a link between the apparent radius of curvature of the universe and a particle whose characteristics are relatively well known, the electron.

In 1974, Dirac published a large numbers hypothesis [12]. He had noted that several ratios of numbers that have the same units oddly always gave certain numbers. We called the greatest of these N [13]. All of the other numbers that he discovered can be deduced from this number by changing its exponent [13].

If we try to find the value of N, many different ways can give the same result. We will do not describe all of them here and we will not try to demonstrate them. However, here are some simple ways of finding N:

- Doing the ratio between the energy E_u contained in the universe and the energy of a photon E_{ph} which would have a wavelength equal to the circumference of the universe.
- Doing the ratio between the apparent mass of the universe m_u and the mass m_{ph} associated to a photon that has a wavelength equal to the circumference of the universe.
- Doing the square of the ratio between the apparent mass of the universe m_u and the Planck mass m_p.
- Doing the square of the ratio between the apparent radius of the universe R_u and the Planck length L_p.
- The most precise and the simplest way remains the equation that uses the inverse of the fine structure constant α raised to the power 57 (see [6]).

$$N = \frac{E_u}{E_{ph}} = \frac{m_u}{m_{ph}} = \left(\frac{m_u}{m_p}\right)^2 = \left(\frac{R_u}{L_p}\right)^2 = \frac{1}{\alpha^{57}} \tag{8}$$

Among all these equalities, we will retain the following one because of its precision:

$$\left(\frac{R_u}{L_p}\right)^2 = \frac{1}{\alpha^{57}} \tag{9}$$

Isolating R_u from this equation, we obtain:

$$R_u = \sqrt{\frac{L_p^2}{\alpha^{57}}} \tag{10}$$

The Planck length L_p comes from the Heisenberg incertitude principle and is defined as being:

$$L_p = \sqrt{\frac{h \cdot G}{2 \cdot \pi \cdot c^3}} \tag{11}$$

According to the CODATA 2010 [8], the Planck constant is evaluated to $h \approx 6.62606957 \pm 0.00000029 \times 10^{-11}$ J·s .

Using the equations **(4)**, **(10)** and **(11)**, we obtain:

$$R_u = \sqrt{\frac{h \cdot r_e}{2 \cdot \pi \cdot m_e \cdot c \cdot \beta \cdot \alpha^{37}}} \approx 1.283107882 \pm 0.000000029 \times 10^{26}\,\text{m} \quad \textbf{(12)}$$

Knowing that:

$$m_e \cdot c^2 = \frac{h \cdot c \cdot \alpha}{2 \cdot \pi \cdot r_e} \quad \textbf{(13)}$$

Isolating r_e , we have:

$$r_e = \frac{h \cdot c \cdot \alpha}{2 \cdot \pi \cdot m_e \cdot c^2} \quad \textbf{(14)}$$

The equation **(12)** then becomes:

$$R_u = \frac{r_e}{\beta^{1/2} \cdot \alpha^{19}} \approx 1.2831078806 \pm 0.0000000081 \times 10^{26}\,\text{m} \quad \textbf{(15)}$$

This equation allows us to obtain the apparent radius of the universe as a function of the classical radius of the electron r_e. The classical radius of the electron r_e is however is not considered as being a fundamental constant in physics since it cannot be measured. Besides, many physicists consider that the true radius of the electron is punctual. The classical radius of the electron can only be calculated. It cannot be obtained from a measurement of an objective reality.

3.6. Value of R_u as a Function of the Charge and the Mass of the Electron

As the classical radius of the electron r_e cannot be considered as a fundamental value in physics since it cannot be directly measured, we suggest here to use directly measurable physical characteristics like the charge q_e and the mass m_e.

The classical radius of the electron r_e is defined as being:

$$r_e = \frac{1}{4 \cdot \pi \cdot \varepsilon_0} \cdot \frac{q_e}{m_e \cdot c^2} \quad \textbf{(16)}$$

Furthermore, the fine structure constant is defined as being:

$$\alpha = \frac{q_e^2}{2 \cdot h \cdot c \cdot \varepsilon_0} \tag{17}$$

Using the equations (16) and (17), the equation (12) becomes:

$$R_u = \frac{q_e^2}{4 \cdot \pi \cdot \cdot \varepsilon_0 \cdot m_e \cdot c^2 \cdot \alpha^{19} \cdot \beta^{1/2}} \tag{18}$$

Knowing that:

$$c = \sqrt{\frac{1}{\mu_0 \cdot \varepsilon_0}} \tag{19}$$

We can rewrite the equation **(18)** as follows:

$$R_u = \frac{\mu_0 \cdot q_e^2}{4 \cdot \pi \cdot m_e \cdot \alpha^{19} \cdot \beta^{1/2}} \approx 1.283107881 \pm 0.000000057 \times 10^{26} \text{m} \tag{20}$$

This equation allows us to obtain the apparent radius of the universe as a function of the charge and the mass of the electron which are two parameters of the electron that are measured with precision. This equation is built from fundamental constants of physics. It could seems logical to believe that this value is the best. However, it is less precise than the equation **(15)**. This may be due to the fact that the value of the classical radius of the electron is obtained from different ways and that the laboratories make a statistical calculation which allows reducing its error. For now, we prefer the equation **(15)** since it shows the smallest error. The situation could change over the time depending on the most recent values of the CODATA [8].

Let's mention that by using the equations **(3)**, **(4)**, **(16)** and **(17)**, it is possible to show that the equations **(12)** and **(20)** are equal to the equation **(7)**. We leave the trouble of verifying this to the reader.

3.7. Value of R_u as a Function of the Rydberg Constant

The Rydberg constant that is known with the highest precision. According to the CODATA 2010 [8], its value is defined as follows:

$$R_\infty = \frac{m_e \cdot q_e^4}{8 \cdot \varepsilon_0^2 \cdot h^3 \cdot c} \approx 10973731.568539 \pm 0.000055 \text{m}^{-1} \tag{21}$$

The speed of light in vacuum c is defined as a function of the relative permeability of vacuum μ_0 and of the relative permittivity of vacuum ε_0:

$$c = \sqrt{\frac{1}{\mu_0 \cdot \varepsilon_0}} \qquad \textbf{(22)}$$

With the help of the equations **(13)**, **(21)** and **(22)**, we obtain:

$$R_\infty = \frac{\alpha^3}{4 \cdot \pi \cdot r_e} \qquad \textbf{(23)}$$

When we isolate the value of the classical radius of the electron r_e from the equation **(23)** and that we put it in the equation **(15)**, we obtain the apparent radius of the universe R_u:

$$R_u = \frac{1}{4 \cdot \pi \cdot R_\infty \cdot \beta^{1/2} \cdot \alpha^{16}} \qquad \textbf{(24)}$$

$$R_u \approx 1.2831078806 \pm 0.0000000068 \times 10^{26}\,\mathrm{m} \qquad \textbf{(25)}$$

This calculation method for R_u seems to be the most precise of all.

3.8. Value of R_u as a Function of the Average Temperature of the Universe T

Since the universe is expanding, its temperature diminishes over time. Consequently, an exact measurement of the average temperature of the cosmic microwave background (CMB) could give indirectly the dimension of the apparent radius of curvature of the universe. So, let’s find an equation that varies mainly as a function of the average temperature T of the CMB.

Let’s start from the following equation that we found in the past [23]:

$$T = \left(\frac{15 \cdot \alpha^2 \cdot h^3 \cdot \beta^4 \cdot c^5 \cdot H_0^2}{8 \cdot \pi^6 \cdot k_B^4 \cdot G} \right)^{1/4} \qquad \textbf{(26)}$$

According to the CODATA 2010 [8], the Boltzmann constant is evaluated at $k_B \approx 1.3806488 \pm 0.0000013 \times 10^{-23}$ J·°K.

Using the equations **(1)**, **(4)**, **(13)** and **(23)**, we obtain the following equation:

$$R_u = \frac{\pi^2 \cdot k_B^4 \cdot T^4}{960 \cdot h^4 \cdot \beta^{13/2} \cdot c^4 \cdot R_\infty^5 \cdot \alpha^{25}} \approx 1.282 \pm 0.032 \times 10^{26}\,\mathrm{m} \qquad \textbf{(27)}$$

This equation is the least precise of all since, for now, the temperature T is not evaluated with much precision. Even if D. J. Fixen [24] has realized CMB measurements and obtained a temperature of $T \approx 2.722548 \pm 0.00057$ °Kelvin, we believe that his result has a bigger error than what he would like to let us believe. We believe that one of the best results obtained until now, is the one given by the space probe Cobra evaluating the average temperature of the CMB at 2.736±0.017 °K in the years 1982 to 1990 [25].

Of course, the obtained precision would be substantially improved by using the average temperature of the CMB that we have already calculated by the past [6].

$$T = \frac{m_e \cdot c^2}{k_B} \cdot \left(\frac{15 \cdot \beta^6 \cdot \alpha^{17}}{\pi^3} \right)^{1/4} \approx 2.736795 \pm 0.000003\,°\mathrm{K} \qquad \textbf{(28)}$$

We then obtain $R_u \approx 1.2831077 \pm 0.0000074 \times 10^{26}$ m.

3.9. Value of R_u as a Function of the Universal Gravitational Constant G

Let's attempt to find here an equation that allows us to determine the apparent radius of curvature of the universe R_u as a function of G. Let's mention here that we make no hypothesis on the constancy or not of G over time or in space.

Let's start from the equation which allows to give the apparent mass of the universe m_u [21,22].

$$m_u = \frac{c^3}{G \cdot H_0} \qquad \textbf{(29)}$$

Let's rewrite this equation using the equation **(7)** and let's isolate R_u:

$$R_u = \frac{G \cdot m_u}{c^2} \qquad \textbf{(30)}$$

We have already shown that the maximum number N of photons (associated with a mass m_{ph}) of wavelength $2\cdot\pi\cdot R_u$ that can be in the universe is [6]:

$$N = \frac{m_u}{m_{ph}} = \frac{1}{\alpha^{57}} \tag{31}$$

Furthermore, the mass m_{ph} can be associated with a photon of wavelength $2\cdot\pi\cdot R_u$:

$$m_{ph} \cdot c^2 = \frac{h \cdot c}{2 \cdot \pi \cdot R_u} \tag{32}$$

We can rewrite the equation **(30)** using the equations **(31)** and **(32)** to obtain this:

$$R_u = \sqrt{\frac{G \cdot h}{2 \cdot \pi \cdot c^3 \cdot \alpha^{57}}} \approx 1.283166 \pm 0.000077 \times 10^{26} \mathrm{m} \tag{33}$$

The indicated value of R_u is the one that we obtain by using the value of G coming from the CODATA 2010 [8]. Of course, it is better if we use the value of G that we obtain from the equation **(4)** (see the recapitulative table at the end of the article). This method is among the least precise ones because of the little precision that is available up to now for the constant G.

4. APPARENT RADIUS OF CURVATURE OF THE UNIVERSE r_u AT OUR POSITION

As already mentioned in the past [3], if the luminous universe is in expansion at the speed of light c [2], it cannot be the same for the material universe [3]. The material universe must necessarily be in expansion at a lower speed than the one of light. Consequently, from our point of view on Earth, we make the hypothesis that the material universe is in expansion at a speed equal to $\beta\cdot c$ with respect to its center of mass. Mathematical calculations have already allowed us to find that $\beta \approx 0.76$ (see the exact equation in **(3)**) [3]. So, the value of r_u is given by the following equation:

$$r_u = \beta \cdot R_u \tag{34}$$

To find r_u, we must find the precise value of R_u. This is exactly what we have done in the previous steps.

The equation **(34)** allows us, using the value of R_u given by the equation **(15)** to conclude that:

$$r_u = \frac{r_e \cdot \beta^{1/2}}{\alpha^{19}} \approx 9.802071983 \pm 0.000000062 \times 10^{25}\,\mathrm{m} \tag{35}$$

In the same way, using the value of R_u from the equation **(24)**, we obtain:

$$r_u = \frac{\beta^{1/2}}{4 \cdot \pi \cdot R_\infty \cdot \alpha^{16}} \approx 9.802071983 \pm 0.000000052 \times 10^{25}\,\mathrm{m} \tag{36}$$

The value of r_u is the one that we find here, at our position in the universe. We are able to calculate indirectly this value by making the hypotheses described in our first document talking on the acceleration of light [3].

5. APPARENT RADIUS OF CURVATURE OF THE HORIZON r_h

As for the surface of a black hole, at the horizon of the universe, the speed of light becomes null because of the index of refraction of the vacuum which tends toward infinite. In this article, we calculated the value of the apparent radius of curvature of the horizon [3], and we obtained:

$$r_h = \frac{2 \cdot G \cdot m_u}{k^2} = \frac{2 \cdot c^3}{k^2 \cdot H_0} \tag{37}$$

The apparent mass of the universe m_u is given by the following equation [3,21,22] :

$$m_u = \frac{c^3}{G \cdot H_0} \tag{38}$$

Using the equations **(1)**, **(37)** and **(40)** we obtain:

$$r_h = \frac{2 \cdot r_e \cdot c^2}{k^2 \cdot \alpha^{19} \cdot \beta^{1/2}} \tag{39}$$

The value k is given by the following equation [3] :

$$k = c \cdot \sqrt{2 + \sqrt{5}} \tag{40}$$

Using the equations **(39)** and **(40)** we obtain:

$$r_h = \frac{2 \cdot r_e}{\alpha^{19} \beta^{1/2} \left(2 + \sqrt{5}\right)} \approx 6.058013646 \pm 0.000000038 \times 10^{25} \text{ m} \quad \textbf{(41)}$$

Using the equations **(23)** and **(41)**, we obtain another equation:

$$r_h = \frac{1}{2\pi \cdot R_\infty \alpha^{16} \beta^{1/2} \left(2 + \sqrt{5}\right)} \approx 6.058013646 \pm 0.000000032 \times 10^{25} \text{ m} \quad \textbf{(42)}$$

This last one is slightly more precise than the equation **(41)**.

6. CONCLUSION

In this document, different methods have been considered to calculate the apparent radius of curvature of the universe R_u. Some are more precise than others. Using the best method that we found, we calculate the apparent radius of curvature r_u of the universe at our position using our factor β. Still using the same value for R_u, we have calculated the apparent radius of curvature of the horizon of the universe.

Lets make a summary table showing the results of R_u obtained with our different calculation methods (see following page).

Thanks to the equations that we found, we see that these parameters can be linked to the characteristics of the electron (its charge, its mass or its classical radius). So, it becomes evident that the infinitely large (the universe) is intimately linked to the infinitely small (to the particles such as the electron) (as Sidharth shows through almost all his book [15]).

Considering that the luminous universe is presently expanding at the speed of light c, the best estimate for R_u that we made in this document will remain valid for about 87 years (from 1983 which correspond to the date when the ″bureau des poids et mesures″ measured the value of the speed of light) before that we had to readjust the most significant digit in the estimated error.

These new equations could, in the future, allow us to show up theoretical links between certain physical phenomena.

Apparent Radius of Curvature of the Luminous Universe R_u		
Equation	Approximate Value	Error
$R_u = \dfrac{1}{4\pi \cdot \alpha^{16} \cdot \beta^{1/2} \cdot R_\infty}$	$1.2831078806 \times 10^{26}$ m	$\pm 0.0000000068 \times 10^{26}$ m
$R_u = \dfrac{r_e}{\beta^{1/2} \cdot \alpha^{19}}$	$1.2831078806 \times 10^{26}$ m	$\pm 0.0000000081 \times 10^{26}$ m
$R_u = \dfrac{c}{H_0}$	$1.2831078807 \times 10^{26}$ m H_0 based on eq. **(1)**	$\pm 0.0000000083 \times 10^{26}$ m
	1.283×10^{26} m H_0 based on Xiaofeng Wang [9]	$\pm 0.016 \times 10^{26}$ m
$R_u = \sqrt{\dfrac{h \cdot r_e}{2\pi \cdot m_e \cdot c \cdot \beta \cdot \alpha^{37}}}$	$1.283107882 \times 10^{26}$ m	$\pm 0.000000029 \times 10^{26}$ m
$R_u = \sqrt{\dfrac{G \cdot h}{2 \cdot \pi \cdot c^3 \cdot \alpha^{57}}}$	$1.283107880 \times 10^{26}$ m G based on eq. **(4)**	$\pm 0.000000043 \times 10^{26}$ m
	1.283166×10^{26} m G based on CODATA [8]	$\pm 0.000077 \times 10^{26}$ m
$R_u = \dfrac{\mu_0 \cdot q_e^2}{4\pi \cdot m_e \cdot \alpha^{19} \cdot \beta^{1/2}}$	$1.283107881 \times 10^{26}$ m	$\pm 0.000000057 \times 10^{26}$ m
$R_u = \dfrac{\pi^2 \cdot k_B^2 \cdot T^4}{960 h^4 c^4 R_\infty \alpha^{25} \beta^{13/2}}$	1.2831077×10^{26} m T based on eq. (28)	$\pm 0.0000074 \times 10^{26}$ m
	1.282×10^{26} m T based on the Cobra space probe [25]	$\pm 0.032 \times 10^{26}$ m

Here are the 2 best results for the apparent radius of curvature of the universe to our position r_u based on the above table:

Apparent Radius of Curvature of the Universe at our Position r_u		
Equation	Approximate Value	Error
$r_u = \beta \cdot R_u = \dfrac{\beta^{1/2}}{4\pi \cdot \alpha^{16} \cdot R_\infty}$	$9.802071983 \times 10^{25}\,\mathrm{m}$	$\pm\, 0.000000052 \times 10^{25}\,\mathrm{m}$
$r_u = \beta \cdot R_u = \dfrac{r_e \cdot \beta^{1/2}}{\alpha^{19}}$	$9.802071983 \times 10^{25}\,\mathrm{m}$	$\pm\, 0.000000062 \times 10^{25}\,\mathrm{m}$

Here are the 2 best results for the apparent radius of curvature of the horizon r_h based on the first table:

Apparent Radius of Curvature of the Horizon of the Universe r_h		
Equation	Approximate Value	Error
$r_h = \dfrac{1}{2\pi \cdot R_\infty \alpha^{16} \beta^{1/2}\left(2+\sqrt{5}\right)}$	$6.058013646 \times 10^{25}\,\mathrm{m}$	$\pm\, 0.000000032 \times 10^{25}\,\mathrm{m}$
$r_h = \dfrac{2 \cdot r_e}{\alpha^{19} \beta^{1/2}\left(2+\sqrt{5}\right)}$	$6.058013646 \times 10^{25}\,\mathrm{m}$	$\pm\, 0.000000038 \times 10^{25}\,\mathrm{m}$

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